Mild solutions and related solution concepts

We consider the stochastic evolution equation

\[ dX_t = (AX_t + F_t(X_t))dt + B_t(X_t)dW_t, \]  

(1)

where \( W \) is cylindrical Brownian motion on a separable Hilbert space \( U \), \( A \) is the generator of a strongly continuous semigroup on a separable Hilbert space \( H \), and \( F \) and \( B \) are nonlinear mappings on appropriate spaces.

a) Assuming that \( F \) and \( B \) vanish, describe the relation between mild solutions of (1) and solutions of the abstract Cauchy problem \( dX_t/dt = AX_t \) [EN99, Section II.6].

b) Assuming that \( B \) vanishes, \( F \) depends only on time, and \( H \) is finite-dimensional, show that the definition of mild solutions of (1) coincides with the variation of constants formula for ordinary differential equations [Arn92, Section 29].

c) Assuming that \( F \) vanishes and \( B \) is constant, show that the notion of mild solutions of (1) corresponds to the integral representation of Ornstein-Uhlenbeck processes (see [Jac96] for definitions and a historical context).

References
