Problem 1 (4 Points). Let \( B \) be a Brownian motion and define the \( \mathbb{R}^2 \)-valued process \( X \) by \( X_i(t) = (\sqrt{X_i} + B(t))^2 \) for \( i = 1, 2 \), and for some \( x \in \mathbb{R}^2 \) such that \( X \) satisfies
\[
\begin{align*}
  dX_1 &= dt + 2\sqrt{X_1}dW, \\
  dX_2 &= dt + 2\sqrt{X_2}dW, \\
  X(0) &= x
\end{align*}
\]
Is \( X \) an affine process?

Problem 2 (4 Points). Compute the characteristic function of \( X(t) \) and verify your result concerning the (supposed) affine property of \( X \).

Problem 3 (4 Points). Let \( b, \sigma > 0 \) and \( \beta \in \mathbb{R} \), and consider the affine process
\[
  dX = (b + \beta X)dt + \sigma \sqrt{X}dW, \quad X(0) = x \in \mathbb{R}_+,
\]
with state space \( \mathbb{R}_+ \).
Compute the corresponding system of Riccati equations.

Problem 4 (4 Points). Consider the Riccati differential equation
\[
  \partial_t G = aG^2 + bG - c, \quad G(0,u) = u
\]
where \( a, b, c \in \mathbb{C} \) and \( u \in \mathbb{C} \), with \( a \neq 0 \) and \( b^2 + 4ac \in \mathbb{C} \setminus \mathbb{R} \). Let \( \sqrt{\cdot} \) denote the analytic extension of the real square root to \( \mathbb{C} \setminus \mathbb{R}_- \), and define \( \theta = \sqrt{b^2 + 4ac} \).
Show that the function
\[
  G(t,u) = \frac{2c(e^{\beta t} - 1) - (\theta(e^{\beta t} + 1) + b(e^{\beta t} - 1))u}{\theta(e^{\beta t} + 1) - b(e^{\beta t} - 1) - 2a(e^{\beta t} - 1)u}
\]
is the unique solution of the Riccati differential equation on its maximum interval of existence \( [0, t_+(u)) \).