Recent developments in dependence modeling **2024**

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Exercise 2

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Exercise 1 (4 Points)**.** Let (X_1, Y_1) and (X_2, Y_2) be independent vectors of absolutely continuous random variables with joint distribution functions F_{X_1,Y_1} and F_{X_2,Y_2}, respectively, with common margins $F_X = F_{X_1} = F_{X_2}$ and $F_Y = F_{Y_1} = F_{Y_2}$. Let C_1 and C_2 denote the copulas of (X_1, Y_1) and (X_2, Y_2), respectively. Show that

$$\begin{aligned} &P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0] \\ &= 4 \int_{[0,1]^2} C_2(u, v) dC_1(u, v) - 1 =: Q(C_1, C_2). \end{aligned} \tag{1}$$

Exercise 2 (4 Points)**.** Let (X, Y) be a vectors of absolutely continuous random variables. Use the definition of Q in (1) to show that

1. the population version of Kendall's τ is given by $\tau(X, Y) = Q(C, C)$ if C is a copula for (X, Y) and
2. the population version of Spearman's ρ is given by $\rho(X, Y) = 3Q(C, \Pi)$, if C is a copula for (X, Y) and Π describes the independence copula $\Pi(u, v) = uv$.

Exercise 3 (4 Points)**.** Show that $3Q(C, \Pi) = 12 \int_0^1 \int_0^1 (C(u, v) - uv) du dv$. Then show that Spearman's rho is a measure of concordance.

Exercise 4 (4 Points)**.** Show that the Average Value at Risk is a law-invariant, coherent risk measure that is also comonotonic additive.