1. **A discrete market**
Let $(\Omega, \mathcal{F}, \mathbb{P})$ be given by $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $\mathcal{F} = 2^\Omega$ and any strictly positive reference measure $\mathbb{P}$. Consider the following one-period market model

$$\bar{S_0} = (1, 5), \quad \bar{S_1}(\omega_1) = (1, 3), \quad \bar{S_1}(\omega_2) = (1, 5), \quad \bar{S_1}(\omega_3) = (1, 7).$$

(a) Show that the market is free of arbitrage. Determine the set of equivalent martingale measures. (3)

(b) Let $C := (S_1^1 - 4)^+$. Does there exist a (self-financing) strategy $\bar{\xi}$ such that $C = \bar{\xi} \cdot \bar{S}_1$? (2)

(c) Compute the set $\{\mathbb{E}^{\mathbb{Q}}[C] \colon \mathbb{Q} \text{ equivalent martingale measure}\}$. (2)

Points for Question 1: 7

2. **Martingale (Transform)**
Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. Let $T \in \mathbb{N}$, $(X_t)_{t=0,\dots,T}$ i.i.d. with $X_t \in L^1(\mathbb{P})$. Define the process $(S_t)_{t=0,\dots,T}$ by

$$S_t := X_0 + \sum_{1 \le k \le t} X_k \, .$$

Show that

(a) $\sigma(S_0, \dots, S_t) = \sigma(X_0, \dots, X_t)$ for $t \geq 0$. (3)

(b) The process (M_t) defined by (2)

$$M_t := S_t - (t+1)\mathbb{E}[X_1]$$

is a martingale.

Points for Question 2: 5

You can achieve a total of **12** points for this sheet.