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http://www.stochastik.uni-freiburg.de/lehre/2015WiSe/inhalte/2015WiSeStochProz

Exercise 11

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Problem 1 (4 Points)**.** Let $g : [0, \infty) \to [0, \infty)$ be strictly increasing with $g(0) = 0$, and let $N = (N_t)_{t\geq 0}$ be a Poisson process with intensity 1, and $M = (M_n)_{n=0,1,\dots}$ is a discrete time Markov chain with values in $\mathbb{Z}$ and transition matrix $\Pi = (\pi_{ij})_{i,j\in\mathbb{Z}}$. Furthermore, N and M are independent.
Show that $X = (X_t)_{t\geq 0}$ with

$$X_t := M_{N_{g(t)}}$$

is a Markov process with respect to the natural filtration and determine the transition kernel and the transition operator.

Hint: Use the Chapman-Kolmogorov equation (Corollary 16.16, see the Skript) and Theorem 16.17 "Existence of Markov processes".

Problem 2 (4 Points)**.** (*a*) Under what conditions (with respect to g and Π) is the process $(M_{N_{g(t)}})_t$ homogeneous.

(*b*) Determine the generator of $(M_{N_{g(t)}})_t$.

Problem 3 (4 Points)**.** Recall from Exercise 10: Let $(X_t)_{t\geq 0}$ be a Brownian motion and

$$Y_t = e^{-t/2} X(e^t - 1).$$

Show that $(Y_t)_{t\geq 0}$ is homogeneous, i.e., $(P_{s,t}f(x) = P_{0,t-s}f(x))$ with generator

$$G^Y f(x) = -\frac{x}{2} f'(x) + \frac{1}{2} f''(x)$$

for $f \in C^2(\mathbb{R})$.

Problem 4 (4 Points)**.** Show that every Feller process with right-continuous paths satisfies the strong Markov property.