Capital requirements, the option surface, market, credit and liquidity risk

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Abstract

The Sato process model for option prices is expanded to accomodate credit considerations by incorporating a single jump to default occuring at an independent random time with a Weibull distribution. Explicit formulas for bid and ask prices are derived. Liquidity considerations are captured by movements in these prices reflecting possible changes in the set of zero cost cash flows acceptable to the market. Capital requirements supporting a trade are taken to be given by the difference between the ask and bid prices. From this perspective of variations in required capital it is observed that the Lehman bankruptcy was primarily a liquidity event for the remaining banks. Further, we observe that variations in capital requirements over time are primarily explained by movements in the option surface and the levels of liquidity with credit variations playing a part occasionally.

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1 Introduction

Risk in the market for derivatives has a number of dimensions of interest to those seeking to regulate these markets. In general apart from price movements we have the risk of movements in the option surface like the volatility, the volatility of volatility, the skewness of the risk neutral distribution and the term structure of at-the-money volatility. Additionally there are the risks of changing liquidity along with variations in the credit standing of the underlying security also embedded in the prices of derivatives. From a regulatory standpoint it is critical that capital requirements be set in a risk sensitive manner with a view to counteracting adverse risk incentives inherited by limited liability contracts embedded in the derivatives world of potentially unbounded liabilities, as argued for example in Madan (2009), and Eberlein and Madan (2010). It is therefore imperative that we understand how all these varied risk dimensions impact derivative capital requirements and how in particular they behaved during the crisis, presumably peaking at the date of the Lehman bankruptcy.

With these objectives in mind we follow the theory of conic finance as set out in Cherny and Madan (2010) to define analytically the bid and ask prices of two price markets. There is an established theory for the bid and ask prices observed in the relatively liquid markets like the markets for stocks that model the inventory and asymmetric information costs of market makers.

In this regard we cite Copeland and Galai (1983), Easley and O'Hara (1987), Glosten and Milgrom (1985). Ahimud and Mendelson (1980), Demsetz (1968), Ho and Stoll (1981, 1983) and Stoll (1978) focus particularly on the order processing and inventory costs of liquidity providers. There have also been numerous statistical studies on the bid ask spread (Roll (1984), Choi, Salandro and Shastri (1988), George, Kaul, and Nimalendran (1991), and Stoll (1989)). In particular Huang and Stoll (1997) consider decomposing the spread into order processing, inventory and adverse selection components. These are not the spreads that are modeled in the two price markets of conic finance as they are the spreads associated with the provision of liquidity in liquid markets.

Yet another approach to spreads in the literature is the introduction of transaction costs (Constantinides (1986), Jouini and Kallal (1995), Lo, Mamaysky and Wang (2004)). The spread now reflects the commission charges of trading and may be related to various empirical aspects of the asset in question including the order flow and the trading volume. These studies also address the costs of trading in relatively liquid markets.

On the other hand there is a large segment of financial markets that creates financial products using the relatively liquid markets for hedging. These are the markets for over the counter structured products. Most transacting is infrequent and one typically buys from a provider at the ask price typically unwinding by selling back to the provider at a substantially reduced bid price. The spreads here are not related to inventory considerations as both parties generally hold the positions out to an explicitly stated contract maturity. In the two price markets of conic finance the focus of attention is not on a spread around a single risk neutral price at which one may in principle trade in both directions at the same price, but shifts to modeling the two separate prices at which transactions occur.

The bid price is now seen as a minimal conservative valuation such that the expected outcome will safely exceed this price under numerous alternative valuation possibilities. Similarly an ask price is a maximal valuation ensuring that the expected payout will fall below the price under a similar set of alternative valuation possibilities. The spreads of conic finance are then tied to the specification of the set of valuation possibilities being entertained. A positive expectation under all the valuation possibilities defines the set of risks acceptable to the market, seen now as a passive counterparty to all financial transactions. Agents are not modeled as trading with each other but just with the market that has no views or preferences but merely tests every proposed transaction for acceptability using its set of valuation possibilities. Conic finance provides us with a formal model of the market that differs from the classical one associated with the law of one price where all transactions with a positive expectation under the single market pricing kernel are accepted. This is a half space of acceptable risks that is replaced in conic finance by a proper cone containing the nonnegative cash flows.

Given theoretically these prices we follow Madan and Schoutens (2010), and Carr, Madan and Vicente Alvarez (2010) and define capital reserves for derivative liabilities as the difference between the ask and bid prices. Theoretically a liability could be unwound by buying it back from the market at the ask price and holding reserves at this level would be quite safe. But it would also be quite a substantial amount of capital that allows no use of funds on taking on the liability because a capital reserve set at the ask exceeds any possible price. Madan and Schoutens (2010) and Carr, Madan and Vicente Alvarez (2010) argue for releasing a conservative valuation like the bid price and holding just the difference in reserves. Assuming that this bid price could be recovered one could couple this with the reserve to cover the unwind at the required ask.

When trust disappears in the market potential transactions have to pass a more stringent collection of tests to be approved. This situation is analytically captured by expanding the set of valuation or test measures under which a positive expectation is being demanded. As a consequence bid prices fall, ask prices rise, and there is a resulting expansion of capital requirements limiting economic activity. The two prices of conic finance attempt to calibrate trust in the market place by explicitly modeling the cone of acceptable risks.

Liquidity risk is then captured by movements in trust as reflected by the cone of risks acceptable to the market. The market is seen as reducing the set of classically acceptable risks defined by a positive risk neutral expectation, by additionally requiring a positive expectation under a whole host of additional valuation possibilities as well. The existence of these additional valuation possibilities introduces the two prices of conic finance and liquidity issues. The original risk neutral measure does all the pricing of classical risks using the linear pricing rule induced by the risk neutral measure. Liquidity risk pricing is nonlinear as the two prices are seen as infima and suprema of a set of valuations making the measure change attaining the two prices dependent on the cash flow being priced and hence the nonlinearity.

For a candidate classical risk neutral measure for the option surface we synthesize the risks by the four parameter model of the Sato process introduced in Carr, Geman, Madan and Yor (2007), that is based on the variance gamma law (Madan and Seneta (1990), Madan, Carr and Chang (1998)) at unit time . The Sato process was shown in Carr, Geman, Madan and Yor (2007) to be particularly effective in synthesizing over a hundred options on numerous underliers at a point of time by four parameters. The model is a one dimensional Markov model and in the absence of static arbitrage there must exist such a model (Carr and Madan (2005), Davis and Hobson (2007)). Hence we employ it as an adequate summary of the option surface at a point of time.

In addition to the risks of movement in the underlying price and the risk neutral parameters describing these probabilities we wish to simultaneously synthesize movements in credit and liquidity risk. In this regard we note that traditionally credit and liquidity have been empirically analysed by looking for securities with the same credit exposure and different liquidities with any remaining price differences being then attributed to the liquidity differences Ahimud and Mendelson (1991) or by controlling for liquidity differentials in the estimation of credit exposures (Tibor, Jarrow and Yildrim (2002)). There are few models parameterizing both aspects in the same model that then allows the estimation to sort out the relative impacts. Furthermore liquidity and credit issues have primarily been studied in the market for stocks and bonds. With regard to options Cetin, Jarrow, Protter (2004) and Cetin, Jarrow, Protter and Warachka (2006) consider liquidity costs inherited by options markets when the market for the underlying asset is not liquid. As a result liquidity costs are incurred by the hedge. We also cite in this connection Cetin, Soner and Touzi (2007). For the approach taken here the underlying asset remains liquid but as we do not have the possibility of complete replication, bid and ask price spreads reflect charges for the need to hold residual risk. Additionally the formulation presented here simultaneously addresses both credit and liquidity issues in the market for derivatives with the focus on capital reserves in the place of pricing or valuation.

As Cetin, Jarrow, Protter and Warachka (2006) write,

"*Risk management is concerned with controlling three financial risks: market risk, credit risk and liquidity risk. Starting with the Black Scholes-Merton option pricing formula, both market and credit risk have been successfully modeled with Duffie (1996) and Bielecki and Rutkowski (2002) offering excellent summaries of these literatures. In contrast, our understanding of liquidity risk is still preliminary.* "

We therefore seek to first extend the Sato process model to accomodate credit risk. In this direction there is already a substantial literature and we cite for example Davis and Lischka (2002), Andersen and Buffum (2003), Albanese and Chen (2005), Linetsky (2006), Atlan and Leblanc (2005) and Carr and Madan (2009) that allows in particular for linkages between comovements in the underlying asset price and the probability of the credit event. In this paper that is an initial foray into jointly modeling both credit and liquidity risk we take a first order approach to credit risk by allowing for its mere existence but ignoring issues of comovements that may now exist in principle in all the three dimensions of market, credit and liquidity. Extensions addressing and then modeling aspects of comovement are here left for future research. With regard to both credit and liquidity we merely allow for existence. We therefore employ a very simple model for the credit event and use a Weibull distribution for an independent time to default.

As already noted both the study of market and credit risk are substantially advanced and we have much to borrow from, making some particular choices suitable to the context. The study of liquidity risk is relatively preliminary but it is fairly widely acknowledged that these risks are at play when spreads are significant enough to deter for example high frequency trading in the associated assets. The bid and ask spreads in stocks and a variety of fixed income securities that have many market participants employing high frequency trading strategies may well be related to the various market maker considerations modeled in the literature for such spread analysis, but as already noted these are the spreads of the liquid markets. Many financial contracts are traded outside such markets where the two prices are just that, the prices for buying from or selling to the market and we then need a theory for such two price markets. Option markets are probably in between these extremes with some liquidity but yet with many shorter maturity out of the money positions being held to maturity.

We adopt the theory of two price markets proposed in Cherny and Madan (2010) and apply it here to the test case of option markets, treating it for the purposes of this paper as a proper two price market thereby ignoring the little liquidity that it does have. Proper two price markets would include the whole host of structured products that are now an established part of the financial markets. However, the structured product markets lack access to published price data. We are therefore employing option markets as a proxy for the two price markets studied in Cherny and Madan (2010). In such two price markets liquidity risk moves away from the law of one price and the associated linear pricing rule, to a nonlinear pricing rule for liquidity risk. Liquidity risk is therefore fundamentally different from market and credit risk as the latter two fall within the classical domain of a linear pricing rule. This observation may help explain the difficulties associated with modeling liquidity risk as an analysis of liquidity may require a paradigm shift in our approach to pricing, viz. a theory for two price markets.

Combining our three considerations of market, credit and liquidity risk we obtain a model yielding closed forms for bid and ask prices with four parameters that synthesize the option surface. Furthermore, the Weibull distribution provides two credit parameters in the expected life or scale of time to default and the shape parameter yielding the sensitivity of the hazard rate to the firm's age. Market and credit risk are modeled within the classical purview of a linear pricing rule. Finally we introduce two parameters capturing movements in the cone of acceptable risks that may be termed the levels of risk aversion and the absence of gain enticement. These are the liquidity parameters of the model yielding nonlinear pricing models for the two prices of bid and ask. In all there are eight parameters in the full model.

We then go on to employ the perspective of two price markets to study capital requirements and how they respond to volatility, the movements of the option surface, credit considerations and the newer modeling of movements in the cone of acceptable risks.

The eight parameters are estimated on data for bid and ask prices for options on four financial firms with sufficient data in the selected period. The estimation is conducted every three days for three years beginning October 23, 2007 and ending September 22, 2010. Capital requirements are then assessed for a variety of options each day and we present an analysis of the contributions of the various risk sources to variations in required capital reserves.

The outline of the rest of the paper is as follows. Section 1 presents the modification of the stock price model which is driven by a Sato process to accomodate an exposure to default. Section 2 briefly describes the computation of bid and ask prices for a cone of acceptable risks defined via concave distortions. Section 3 presents some stylized facts about how the various parameters impact capital requirements. Section 4 presents the data and estimation results for three years on four financial firms. Section 5 decomposes changes in required capital into the various risk components around the Lehman bankruptcy. Section 6 presents a time series for the total and relative contributions to capital activity of the three broad sources of risk, the option surface, liquidity and credit. Section 7 concludes.

2 Accomodating default in derivative pricing

We begin with a brief review of a successful four parameter model that calibrates well the lower maturity option surfaces on major indices that may be viewed as free of default. This is the Sato process model first introduced by Carr, Geman, Madan and Yor (2007). The starting point for the construction of this model is a self decomposable law for the risk neutral distribution of the logarithm of the stock price at unit time that we take to be a year.

Self decomposable laws were studied by Lévy (1937) and Khintchine (1938) and are defined by the property that a random variable X is self decomposable if for every real c, $0 < c < 1$, there exists a random variable $X^{(c)}$ independent of X such that

$$X \overset{(d)}{=} cX + X^{(c)}.$$

Lévy (1937) and Khintchine (1938) showed that the self decomposable laws are the class of limit laws. More exactly these are the laws of limits of sequences of sums of independent random variables appropriately scaled and centered. The self decomposable laws form a proper subclass of the class of infinitely divisible laws with a special structure to their Lévy measures $k(x)dx$. In particular the function $|x|k(x)$ must be decreasing for $x > 0$ and increasing for $x < 0$. An example of such a law is given by the variance gamma process at unit time and in this case $|x|k(x)$ has the form $\exp(ax - b|x|)$ for $|a| < b$, and we clearly have the required property.

Sato (1991, 1999) showed that one may associate with such a self decomposable law at unit time a process with independent but inhomogeneous increments by defining the marginal laws of the process at time points t upon scaling the law at unit time. Hence we have that

$$X(t) \overset{(d)}{=} t^{\gamma} X, \ t > 0.$$

Sato constructed the precise representation for $X(t)$ as an additive process.

Consider T such that for $t < T$, $X(t)$ has a finite time zero exponential moment. Then define $\omega(t)$ by

$$\exp(-\omega(t)) = E_0 \left[\exp \left(X(t) \right) \right].$$

Carr, Geman, Madan and Yor (2007) defined a positive stock price process $S(t)$ with rate of return equal to $r - q$ for an interest rate r and a dividend yield q by

$$S(t) = S(0) \exp((r - q)t + X(t) + \omega(t)).$$

They showed that this simple normalized exponential of an additive process calibrates option surfaces quite well. It is also known that the Lévy process associated with the random variable X at unit time fails to fit the option surface as it has a too fast paced reduction in skewness and excess kurtosis, when compared to model free estimates of these quantities from market data (Konikov and Madan (2002)).

When X is the variance gamma law at unit time with its classical representation as a scaled Brownian motion $W(t)$ with drift, time changed by a gamma process $g(t;\nu)$. Specifically,

$$X(t; \sigma, \nu, \theta) = \theta g(t;\nu) + \sigma W(g(t;\nu))$$

wherby we have a three parameter model, with parameters σ, the volatility of the scaled Brownian motion, ν, a volatility of volatility or the variance rate of the gamma time change, and θ the drift of the underlying Brownian motion that controls the skewness. The gamma process is an increasing pure jump Lévy process with independent identically distributed increments over regular nonoverlapping intervals of length h that are gamma distributed with density $f_h(g)$ where

$$f_h(g) = \frac{g^{\frac{h}{\nu}-1} e^{-\frac{g}{\nu}}}{\nu^{\frac{h}{\nu}} \Gamma\left(\frac{h}{\nu}\right)}, \ g > 0.$$

The Sato process constructed from the variance gamma law at unit time $X(1)$ has an additional scaling parameter γ. In all we have a four parameter model for the option surface. The parameter γ helps to calibrate the term structure of at the money volatility.

We now extend this model in a simple way to merely allow for the possibility of default modeled by a single jump of the stock price to zero. We recognize that numerous formulations in the literature, cited above, already model the hazard rate for the single jump to default as a decreasing function of the stock price.

In the interest of jointly modeling credit and liquidity considerations for the first time we take a simpler model for default. Here the logarithm of the stock price process in the absence of default has an additive Sato specification and we take the hazard rate to be purely deterministic and consistent with a time dependent survival probability given by a Weibull distribution. We employ the Weibull distribution as it is a widely used distribution for life times and default times (Lambrecht, Perraudin and Satchell (1997), Lee and Urrutia (1996)). It allows for both increasing and decreasing hazard rates with respect to age.

We thereby write the defaultable stock price process as

$$\widetilde{S}(t) = \widetilde{S}(0)\exp((r-q)t + X(t) + \omega(t))\frac{\Delta(t)}{p(t)}$$

where the process $\Delta(t)$ starts at one and makes a single move by a jump down to zero at an independent random time τ. Note that as $p(0) = 1$ we have $\widetilde{S}(0) = S(0)$.The probability that $\Delta(t)$ is one is

$$p(t) = \exp\left(-\left(\frac{t}{c}\right)^{a}\right)$$

where the parameter c controls the scale or average life and the shape parameter a exceeds unity for hazard rates that increase with age, while $a < 1$ otherwise. The expected life is $\Gamma(1+\frac{1}{a})c^{\frac{1}{a}}$.

Let $F_t(s)$ be the distribution function of the stock price conditional on no default implied by the distribution of the Sato process at time t. Specifically

$$F_t(s) = P\left(S(t) \leq s\right).$$

The distribution function of the defaultable stock price at time t then is

$$\begin{aligned}
\widetilde{F}_t(s) &= P\left(\widetilde{S}(t) \leq s\right) && (1)\\
&= P\left(Default\ by\ t\right) + P(No\ Default\ by\ t\ and\ S(t) \leq sp(t)\ ) && (2)\\
&= 1 - p(t) + p(t)F_t(sp(t)). && (3)
\end{aligned}$$

We shall see that the bid and ask prices for put and call options are determined completely by the stock price distribution function and we employ equation (3) in these expressions to determine bid and ask prices on call and put options on a defaultable underlier.

We note that credit risk is typically analysed by modeling the probability of default and recovery in default (Lando (2009)). For options the recovery is clear as the call is worthless and the put receives the strike. Credit issues then turn on the probability of default.

At this point we have a six parameter distribution function for the price of a defaultable stock. These are the four option surface parameters $\sigma, \nu, \theta, \gamma$ coupled with the parameters of the Weibull survival function c, a. The pricing is also classical at this point with the specification of a single risk neutral law for the underlying asset.

3 Nonlinear Modeling of Liquidity using the Theory of Two Price Markets

We employ here the principles of two price markets set out in Cherny and Madan (2010). The market is modeled as a passive counterparty and all economic agents may trade with the market, delivering to the market cash flows that are market acceptable. The market accepts at zero cost all nonnegative cash flows, and more generally it accepts a convex cone of cash flows containing the nonnegative cash flows. The theory of two price markets differs from the classical one price theory only by reducing the set of cash flows acceptable to the market at zero cost from the half space of positive alpha trades to a proper convex cone containing the nonnegative cash flows. As already noted there are other ways to model bid and ask prices that focus on the microeconomic concerns of market makers providing the liquidity in highly liquid markets. The theory of two price markets expounded in Cherny and Madan (2010) continues to model the market as a classical passive counterparty with the only change being a dependence of the terms of trade on the trade direction. The dependence is however derived from an exogenous specification for the structure of risks that market participants may deposit in the market at zero cost. They may deposit a cash flow nonnegative to the market, but more generally deposit a convex cone containing such nonnegative cash flows. We believe that an investigation of such a minimal departure from the classical model is worthy of an independent investigation before one personalizes counterparties by bringing in game theoretic considerations into the analysis. The focus on possible losses and how conservative the resulting valuations are then depends on the size of the cone of zero cost acceptable risks that constitutes an important primitive defining the market.

Artzner, Delbaen, Eber and Heath (1999) show that all such cones are defined by a convex set of probability measures $\mathcal{M}$ with the property that X is market acceptable at zero cost if

$$E^Q[X] \geq 0, \text{ for } Q \in \mathcal{M}.$$

The set of test measures $\mathcal{M}$ describe all the valuation measures that must approve the acceptability of a random variable. These measures were refered to as scenario measures in Carr, Geman and Madan (2001). To ensure that the set of acceptable risks is smaller than the classical one given by positive expectation under a single risk neutral measure the set $\mathcal{M}$ should contain a risk neutral measure.

For an operational definition of such cones Cherny and Madan (2010) consider accepting all random variables X with a distribution $F(x) = P(X \leq x)$, provided

$$\int_{-\infty}^{\infty} x d\Psi(F(x)) \geq 0,$$

for some fixed concave distribution function Ψ. The set of test measures or scenario measures in this case consists of measure changes $Z(u)$ on the unit interval $0 \leq u \leq 1$, with respect to the uniform density for $U = F(X)$ such that the antiderivative L, for $L' = Z$, is bounded by the distortion, i.e. $L \leq \Psi$. We denote this set of test measures $\mathcal{M}^{(\Psi)}$.

Cherny and Madan (2010) then show that the bid price $b(X)$ for a cash flow X with distribution function F is given by the acceptability of $X - b(X)$ and

$$b(X) = \int_{-\infty}^{\infty} x d\Psi(F(x)) \quad (4)$$

$$= \inf_{Q \in \mathcal{M}^{(\Psi)}} E^Q[X]. \quad (5)$$

Similarly the ask price $a(X)$ requires the acceptability of $a(X) - X$ and

$$a(X) = -\int_{-\infty}^{\infty} x d\Psi\left(1 - F(-x)\right) \quad (6)$$

$$= \sup_{Q \in \mathcal{M}^{(\Psi)}} E^Q[X]. \quad (7)$$

For the specific cash flows associated with call and put options one obtains on integration by parts specific formulas for the bid and ask prices. The bid and ask prices for calls are denoted $C_b(K,t), C_a(K,t)$ while for puts we write $P_b(K,t), P_a(K,t)$ for a strike K and a maturity t. We then have that

$$\begin{aligned}
C_b(K,t) &= \int_K^{\infty} (1 - \Psi(\widetilde{F}_t(s)))ds \\
C_a(K,t) &= \int_K^{\infty} \Psi(1 - \widetilde{F}_t(s))ds \\
P_b(K,t) &= \int_0^{K} (1 - \Psi(1 - \widetilde{F}_t(s)))ds \\
P_a(K,t) &= \int_0^{K} \Psi(\widetilde{F}_t(s))ds
\end{aligned}$$

The specific distortion we employ is *minmaxvar*2 introduced in Madan and Schoutens (2010) which is given by

$$\Psi(u) = 1 - \left(1 - u^{\frac{1}{1+\lambda}}\right)^{1+\eta}, \lambda > 0, \eta > 0.$$

Here λ controls the rate at which the derivative of Ψ goes to infinity at zero and represents the coefficient of loss aversion in the market, while η controls the rate at which the derivative of the distortion goes to zero at unity and represents the degree of the absence of gain enticement. Expectation under a concave distortion is also an expectation under a measure change where the measure change is given by $\Psi'(F(x))$ and depends on the cash flow being valued via the distribution function. Higher values of λ induce a greater upward reweighting of losses as this raises $\Psi'(u)$ for u near zero where we have losses and hence one may associate higher values of λ with more risk aversion. Higher values of η

on the other hand lower $\Psi'(u)$ for u near unity where we have gains and this reweights gains downwards and so may be associated with a higher absence of gain enticement.

With these two parameters added on we have an eight parameter model for bid and ask prices with the latter two prices being nonlinear as formally the bid is the infimum of valuations while the ask is a supremum of such valuations as per equations (5) and (7).

The parameters λ, η are liquidity parameters for when they are increased the set of acceptable risks is reduced, with bid prices falling and ask prices rising. As a consequence any potential offer to sell at a price above the old market bid must now either take a greater price impact for immediate sale or wait longer for a price recovery. Similarly any potential offer to buy below the old market ask has a greater price impact or waiting time. Liquidity risk is typically seen in such price impact terms (Ericsson and Renault (2006)).

Capital requirements are set by the difference between the ask and bid prices as argued in Carr, Madan and Vicente Alvarez (2010) or Madan and Schoutens (2010). Basically for a liability to constitute an acceptable risk it must be supported by the ask price viewed as the capital or cost of unwinding the position at possibly unfavorable terms. However one gets credit for the bid price as a possible conservative valuation for the position and only the excess need be held in reserve. Note importantly, that the bid and ask prices here are not those associated with the concerns of market makers providing liquidity to markets that trade the associated asset with some high frequency, but rather these are the two prices of a two price economy evaluating conservatively for the acquisition and sale of infrequently traded risks. The difference between the ask and bid prices can also be seen as aggregating what could be lost as an asset with a valuation down to the conservative bid plus what could be lost as a liability with the need to unwind at an unfavorable ask price. The capital reserve is therefore being set with a view towards measuring the possible loss in the contract.

Furthermore from the nonlinear structure of the associated pricing rules of equations (5) and (7) respectively it is clear that a packaged risk has a higher bid and a lower ask than the sum of its components. Hence such price computations should be and would be done at a suitable level of risk aggregation. Most structured products are issued at some level of aggregation in both structure and size of issue. We merely illustrate our computations at the level of data for bid and ask prices of calls and puts. The principles and procedures would in practice be applied at a suitable level of aggregation permitting some netting implicit in the pricing equations.

Given this formulation for reserves, they are then responsive to movements in the option surface parameters $\sigma, \nu, \theta, \gamma$, the credit parameters c, a, as well as the liquidity parameters λ, η. We shall study the relationship between these parameters and capital requirements first in a stylized setting in the next section and then over a three year data period ending September 22, 2010 for the four financial firms Bank of America BAC, Goldman Sachs GS, J.P.Morgan Chase JPM, and Wells Fargo WFC.

4 Capital sensitivity to parameters in a stylized setting

We take as a base setting for the option surface parameters, the mean value of the estimated parameters across time for the four banks, studied later in the paper. For the liquidity and credit parameters we take a stylized value reflecting a symmetric cone with $\lambda = \eta = .1$. The expected life parameter is set at 5 years and the Weibull shape parameter or hazard rate sensitivity is set at 1.25. These are risk neutral parameter values, and CDS prices are typically quoted most actively at five years, though estimates in section 4 later are larger than five years. The use of a five year life is thereby quite conservative. Risk neutral hazard rates using a Weibull density were reported well above 1.25 for example in Madan, Konikov and Marinescu (2006). The base parameter setting is

σ	ν	θ	γ	λ	η	c	a
0.3725	0.6925	−0.3863	0.4724	0.1	0.1	5	1.25

For a portfolio of options we take 10 options with five strikes and two maturities. The maturities are 3 and 6 months. With the spot level set at 100, and zero interest and dividend yield, the strikes are 80, 90, 100, 110 and 120. The options are out of the money, except the option with a strike of 100 is a call . We first report on the gradient of total capital defined as the sum over all options of the difference between the ask and bid prices for these options. The gradient is computed at the base point with respect to each of the eight parameters. This gradient is given by

σ	ν	θ	γ	λ	η	c	a
74.5244	1.3297	−24.2293	−36.6515	205.6706	183.2942	−2.2387	−22.6903

We observe that volatility, σ, and the volatility of volatility, ν, raise capital requirements while an increase in skewness, θ, improves the return distribution and reduces capital. An increase in the volatility spread, γ, or the scaling parameter lowers capital requirements as it reduces volatilities at each maturity below unity, raising them for the longer maturities. Reducing the cone by raising either the coefficient of loss aversion λ, or raising the coefficient for the absence of gain enticement, η, raises capital requirements. On the credit side we see that lowering the expected life raises capital requirements while an increase in the Weibull shape parameter lowers capital requirements as it raises the growth rate of the stock. The actual effect on capital requirements depends critically not just on the gradient but also on the actual change in the parameters.

In order to better appreciate the difference of the effects of changes in liquidity and credit on bid and ask prices we present a graph of the response of bid and ask prices on a 20% out of the money put and call option for an annual maturity of changes in $\lambda = \eta$ and changes in c for a fixed value of $a = 1.25$. We vary c from 1 to 10 years and vary λ from .05 to .2. The other parameters are as in the base case. We present in Figure 1 the effect of the expected life parameter on bid and ask prices.

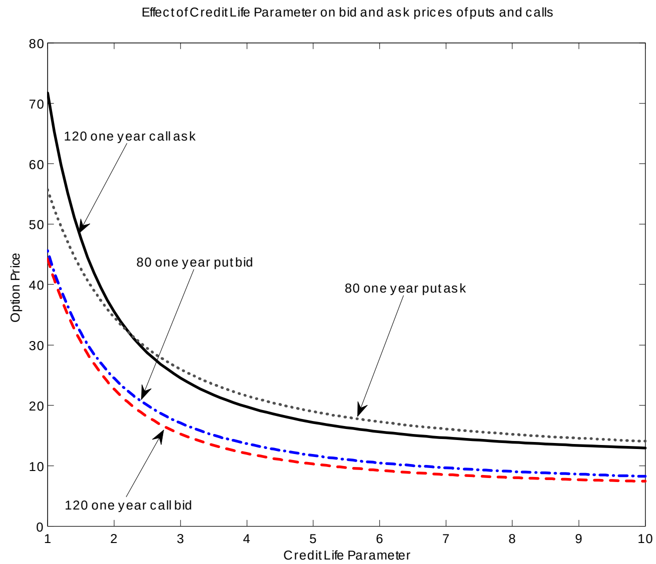

Figure 1: Bid and ask prices for a one year 80 put and a one year 120 call as we vary the expected life parameter

Figure 2 shows the effects on the same options of varying the liquidity parameter.

We clearly see the different effects of variations in credit and liquidity on bid and ask prices of options. While for the former both prices move in the same direction the opposite is the case with respect to variations in liquidity. Hence, credit and liquidity are differentiated economic events.

5 Data and calibration summary

The purpose of the empirical analysis is not to test the proposed model. The adequacy of these models for synthesizing option data has been demonstrated in earlier studies and we cite Carr, Geman, Madan and Yor (2007), Cherny and Madan (2010), Carr and Madan (2009) as examples. There are other models that could be used for this purpose like a jump diffusion model or a Lévy process more generally, but as noted in Carr, Geman, Madan and Yor (2007), Lévy processes do not fit the surface of option prices and it was this failure on the part of Lévy processes that led to the development of the Sato process in the first place. Stochastic volatility models could be used to synthesize the surface but they are at a minimum two dimensional Markov processes and it is well known that option prices at a single time point provide little information on mean reversion and the volatility of volatility. Furthermore as shown in Carr and Madan (2005) and Davis and Hobson (2007), the absence of static arbitrage

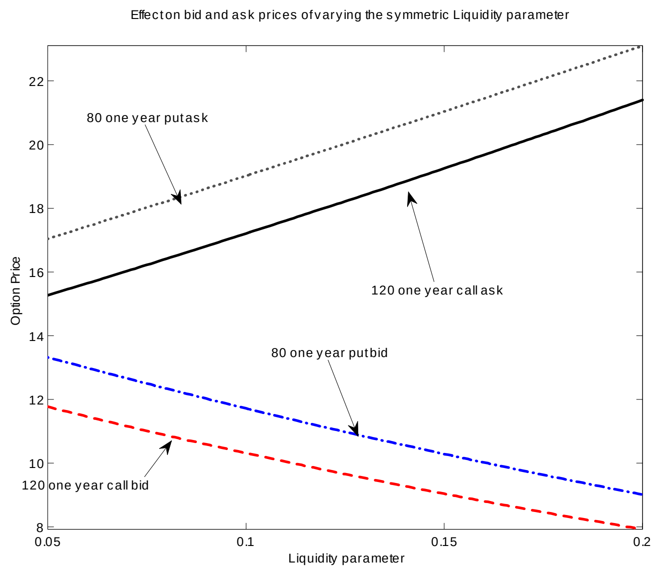

Figure 2: Effect on one year 80 put and one year 120 call of varying the symmetric liquidity parameter.

implies the existence of a one dimensional Markov process synthesizing an option surface at a point of time. The Sato process and its enhancement here is such a process. There is also no issue of in sample or out of sample analysis as the estimation is not conducted over any sample period that constitutes the in sample period. The estimation is on the cross section of prices by strike and maturity on a single day.

The object here is to use a mix of established models synthesizing option surfaces at a point of time to estimate a risk neutral law jointly incorporating for the first time market, credit and liquidity components. The risk neutral law is for a point of time and uses data on option prices at one time point only to evaluate the relative contributions to capital attributable to market, credit and liquidity considerations as embedded in the parameters related to these effects. The measure of capital employed is the difference between the two extreme prices of conic finance as a conservative assessment of loss exposure related to unfavorable unwinds.

The eight parameters of our model are calibrated every third day on bid and ask option prices for three years ending in September 22, 2010 for the four banks, BAC, GS, JPM, and WFC. There were 237 calibrations for each of the four names. Summary statistics for the eight parameter estimates and the corresponding goodness of fit metrics are presented. The goodness of fit metrics are the root mean square error $rmse$, the average absolute error aae, and the average percentage error defined as the average absolute error relative to the average option price in the sample. Also presented are the average number of options used in the calibrations. There are four tables, one for each bank, partitioned into two pieces, one for the parameters and the other for the goodness of fit metrics. Shown are the means, standard deviations and a variety of quantiles for the smoothed parameters and the mean and standard deviations of the goodness of fit statistics.

There were on average 30 to 80 options in the various calibrations. The average percentage error was around 3%. This compares favorably with published and practical experience on such calibrations. Tables 1 through 4 provide the details for the four banks.

We summarize that for BAC the value for σ ranged from .3 to .67 at the 10 and 90 percentile points. The corresponding figures for ν, were .3328, and .9764, for θ, $-0.83, -0.0396$, and for γ, $.3572, .5660$. On the same quantiles loss aversion ranged from 28 to 190 basis points while the absence of gain enticement went from 14 basis points to 84 basis points. The credit parameter related to the average life went from 13 to 41 years, while a ranged from .49 to 4.85.

The comparable statistics for GS were σ, $.3153, .3865$, ν, .4018, 1.0111, $\theta, -.5646$, $-.2148$, and γ, $.3826, .5307$. For loss aversion we have 2 to 155 basis points and gain enticement goes from 20 to 125 basis points. Credit life ranges from 19 to 53 years while a goes from .7349 to 7.6455.

For JPM these values are σ, $.2905, .4488$, ν, .4705, .8939, $\theta, -.6885$, $-.2573$, and γ, $.4020, .5379$. For loss aversion we have 3 to 125 basis points and gain enticement goes from 51 to 161 basis points. Credit life ranges from 13 to 37 years while a goes from .9873 to 4.8703.

Finally for WFC we get σ, $.2161, .4731$, ν, .3950, 1.0302, $\theta, -.8554$, $-.1614$, and γ, $.3609, .5462$. For loss aversion we have 19 to 217 basis points and gain enticement goes from 69 to 267 basis points. Credit life ranges from 12 to 51 years while a goes from .6205 to 4.1181.

Additionally we present in Figures 3 and 4 two graphs of the time series for all the eight parameters. The first graph covers the four option surface parameters for all four banks while the second graph covers the liquidity and credit parameters.

6 Capital requirement movements around the Lehman bankruptcy

In this section we enquire into the nature of the Lehman bankruptcy event for the four banks. For this purpose we set up a hypothetical options book of 10 options consisting of five strikes at each of two maturities. We work with a zero interest rate and dividend yield and the maturities are 3 and 6 months with the spot at 100 and strikes at $80, 90, 100, 110, 120$. The options are out of the money, with the exception of the one with the 100 strike which is a call.

For each of the four banks we consider the calibrated parameters about 3 weeks before the bankruptcy at August 26 2008 and three weeks after the

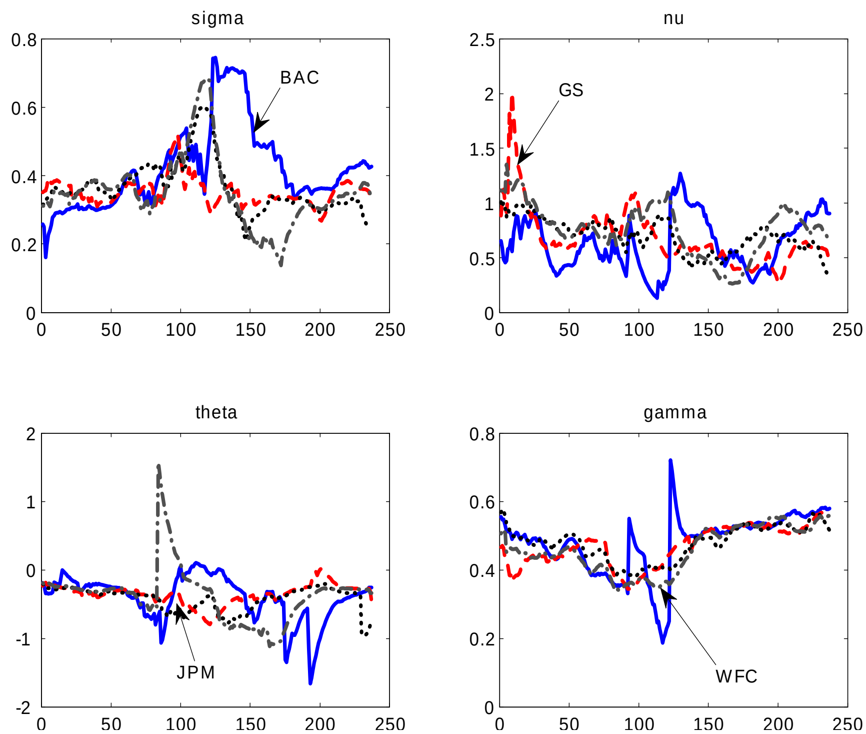

Figure 3: Time series of the four Sato process parameters of volatility σ, volatility of volatility ν, skewness θ and the volatility term structure γ for the four banks. BAC is shown with a solid line, GS with a dashed line, JPM with a dotted dotted line and WFC with a dash dot line.

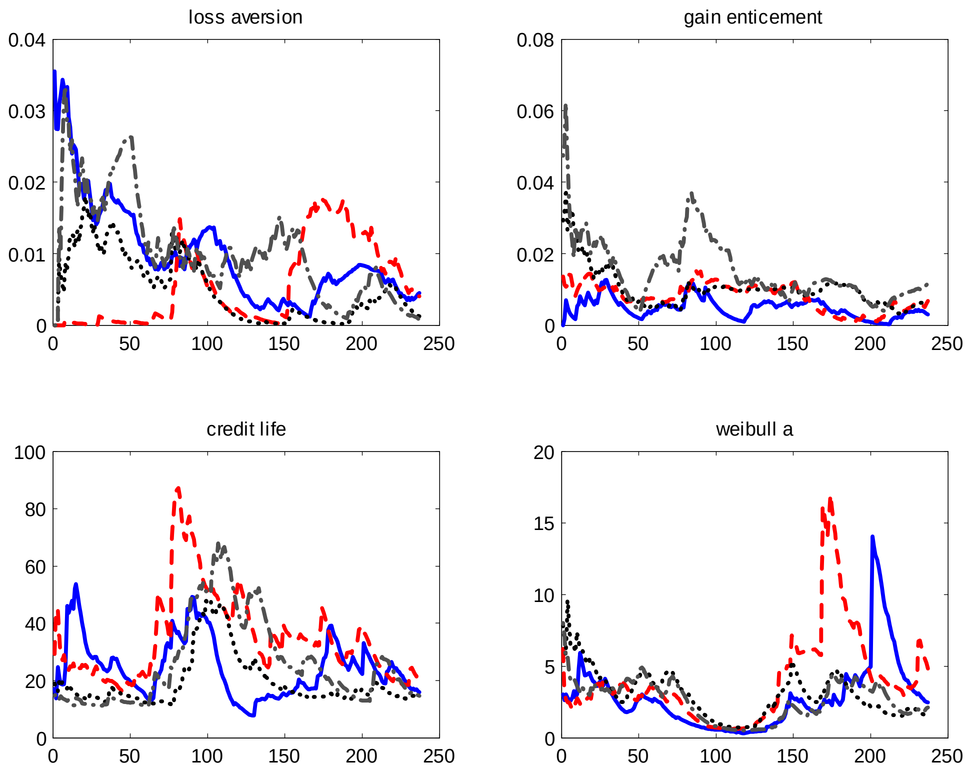

Figure 4: Time series of credit and liquidity parameters, c, a and λ, η for the four banks. BAC is shown with a solid line, GS with a dashed line, JPM with a dotted dotted line and WFC with a dash dot line.

bankruptcy at October 8 2008. For each of the ten options we determine using calibrated parameters the bid and ask prices for these options and the capital required measured as the sum over all 10 options of the difference between the ask and bid prices. This total required capital is computed at the two dates for the four banks and the values are displayed in Table 5 along with the percentage increase.

TABLE 5
Pre and post Lehman capital needs
on the hypothetical portfolio

	BAC	GS	JPM	WFC
Pre Lehman	2.3684	1.1851	2.0325	4.5648
Post Lehman	5.2694	3.8898	4.4995	8.3947
Percentage increase	122.48	228.22	121.38	83.89

These are significant increases in capital requirements at market calibrated stress levels for the cones of acceptable risks. Regulatory settings could even be more conservative than these values.

We now decompose this increase in capital requirements into eight risk sources represented by changes in the eight parameters. The capital increases are computed for hypothetical options on returns with the spot at 100. Any changes in the capital requirements are then due to variations in the parameters. Since capital is now seen as a deterministic function of the form $c = g(\Theta)$ one may approximate the change by

$$\Delta c \approx \left(\frac{\partial g}{\partial \Theta} |_{\Theta_0} \right) \Delta\Theta. \tag{8}$$

We compute the gradient vector at the parameter point for August 26, 2008 and then evaluate the product of the gradient with the change in the parameter value between October 8, 2008 and August 26, 2008. We determine the contribution of each parameter as given by the product of the gradient with respect to the parameter times the change in the parameter. The relative contribution is then obtained on dividing the contribution by the right hand side of equation (8). The relative contributions are given in Table 6 for the four banks.

TABLE 6
Relative parameter contributions to capital requirements
from pre to post Lehman bankruptcy

	BAC	GS	JPM	WFC
σ	0.0406	-0.0657	0.0136	0.1003
ν	0.0254	-0.0026	0.0002	0.0192
θ	0.3476	0.0526	0.0307	-0.0264
γ	0.0409	0.0672	0.0750	0.1077
λ	0.0374	0.8513	0.3998	0.0673
η	0.4854	0.0972	0.4808	0.7318
c	-0.0073	0.0	0.0	0.0
a	0.0299	0.0	0.0	0.0

We see from Table 6 that the major contribution to changes in capital requirements came in this instance from movements in the liquidity parameters. Changes in credit played a minor role. On this evidence it is suggested that the Lehman bankruptcy was primarily a liquidity event and not a credit event for the other large banks.

7 Capital activity and risk contributions across time

We report here on the total and relative contribution to changes in capital requirements of parameter movements. For this purpose we employ the smoothed parameter values at each calibration date. The change in capital attributed to a parameter is measured by the absolute value of the sum over all ten options of the gradient of capital required for the option times the change in the parameter to the next calibration date. The gradient is computed at the calibration date. For the relative contribution we divide these positive parametric contributions by their sum.

We then partition the relative changes into three groups: The option surface, liquidity and credit. The option surface contribution is given by the sum over the contributions of $\sigma, \nu, \theta, \gamma$. For liquidity we sum the contributions of λ, η while for credit we sum the contributions of c, a. Figures 5, 6, 7 and 8 show the total and relative contributions of the three sets of risks on capital activity for the four banks through the three years ending September 22, 2010.

From these figures it is obvious that liquidity and the option surface are the major contributors to variations in required capital with credit occasionally playing a part.

8 Conclusion

The Sato process model for option prices introduced in Carr, Geman, Madan and Yor (2007) is expanded to accomodate credit considerations by incorporating a single jump to default occuring at an independent random time with a Weibull distribution. Following Cherny and Madan (2010) explicit formulas for bid and ask prices are constructed. Movements in these prices are then seen as synthesizing the effects on required capital of changes to the option surface, credit and liquidity. The final model for bid and ask prices of put and call options has eight parameters, four from the Sato process, two from the credit market and the underlying Weibull distribution and two for liquidity.

This eight parameter model is estimated on daily option pricing data for four banks over a three year period ending September 22, 2010. We follow Carr, Madan and Vicente Alvarez (2010) and define capital requirements supporting a trade by the difference between the ask and bid prices. From the perspective of variations in capital required measured with respect to a hypothetical portfolio for options on returns it is observed that the Lehman bankruptcy was primarily

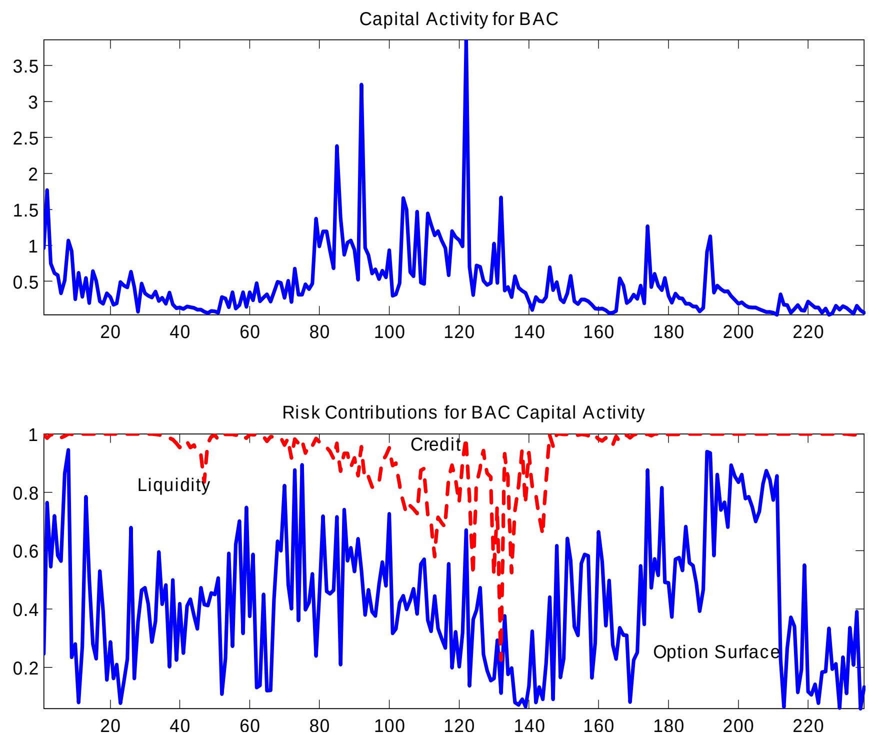

Figure 5: The total absolute change in required capital between successive calibration dates due to movements in all parameters and the relative contributions of changes due to the options surface, credit and liquidity. The first panel presents the total change. The second panel displays the contribution of the option surface below the solid line. Above the dashed line is the contribution of credit while liquidity has the contribution between the two lines. The results are for BAC.

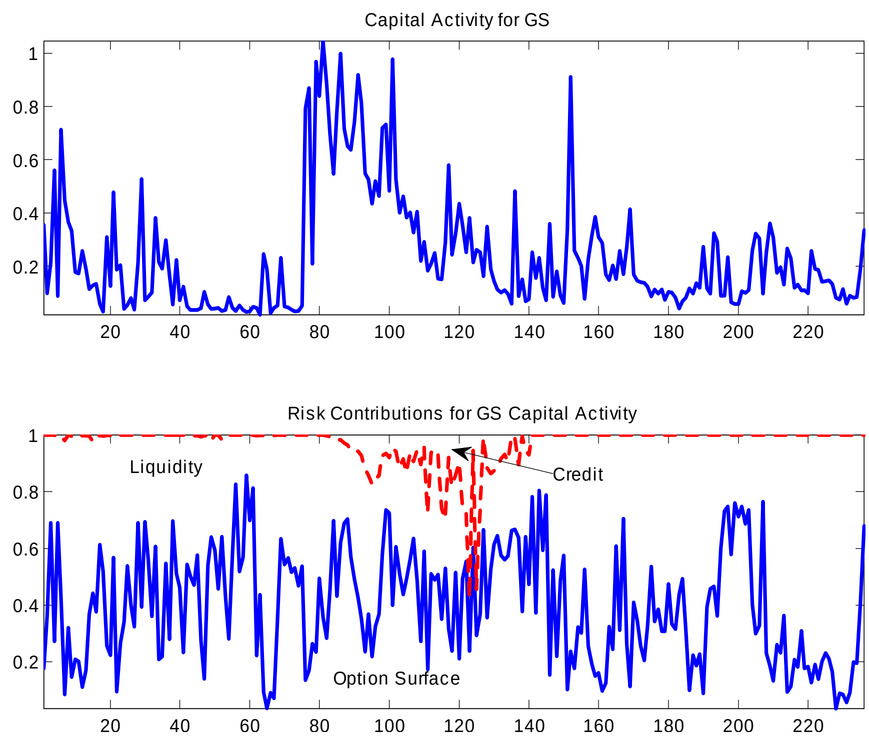

Figure 6: The total absolute change in required capital between successive calibration dates due to movements in all parameters and the relative contributions of changes due to the options surface, credit and liquidity. The first panel presents the total change. The second panel displays the contribution of the option surface below the solid line. Above the dashed line is the contribution of credit while liquidity has the contribution between the two lines. The results are for GS.

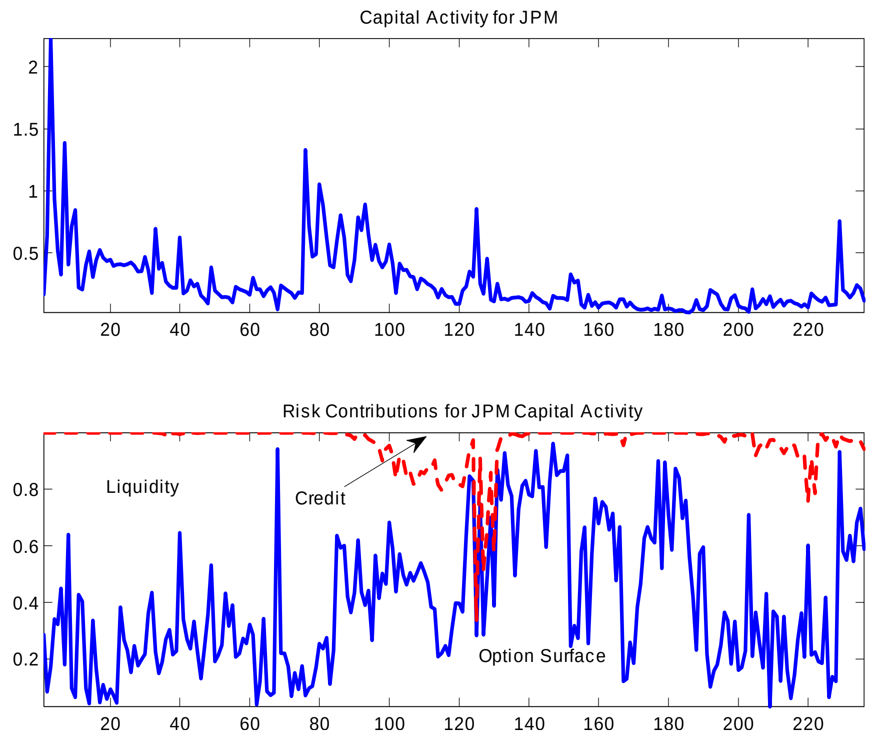

Figure 7: The total absolute change in required capital between successive calibration dates due to movements in all parameters and the relative contributions of changes due to the options surface, credit and liquidity. The first panel presents the total change. The second panel displays the contribution of the option surface below the solid line. Above the dashed line is the contribution of credit while liquidity has the contribution between the two lines. The results are for JPM.

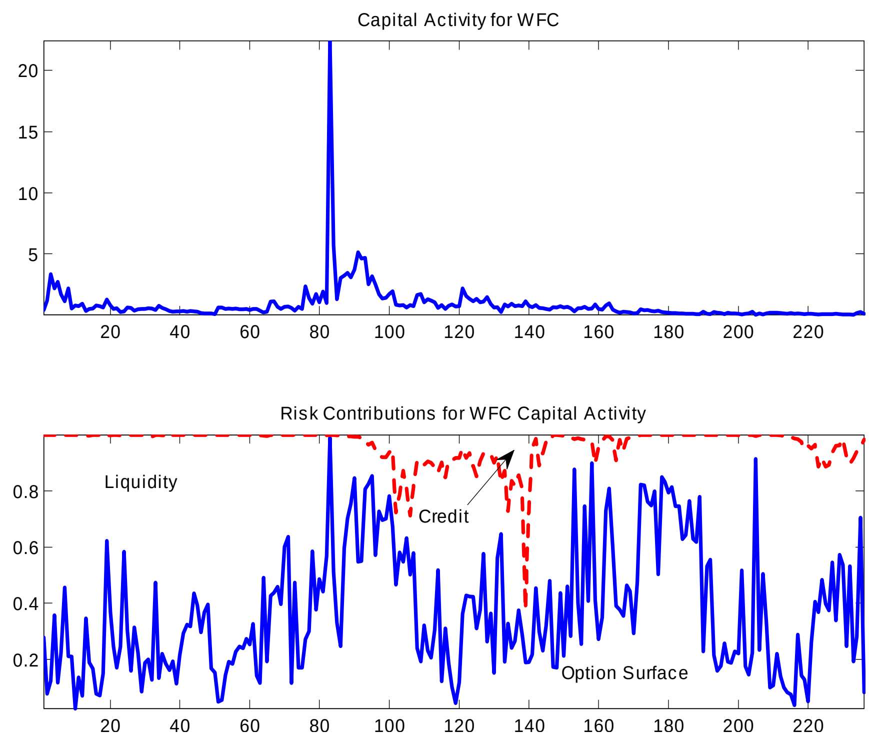

Figure 8: The total absolute change in required capital between successive calibration dates due to movements in all parameters and the relative contributions of changes due to the options surface, credit and liquidity. The first panel presents the total change. The second panel displays the contribution of the option surface below the solid line. Above the dashed line is the contribution of credit while liquidity has the contribution between the two lines. The results are for WFC.

a liquidity event for the other banks. We also observe that for explaining the variation in capital requirements over time, a major role is played by the option surface and liquidity considerations with credit playing a part occasionally.

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TABLE 1

Calibration Results on BAC of Credit and Liquidity Modified VGSSD Sato process

	Option Surface Parameters				Liquidity Parameters		Credit Parameters	
	sigma	nu	theta	gamma	lambda	eta	c	a
Mean	0.4202	0.6275	-0.4038	0.4802	0.0099	0.0048	24.6438	2.8165
Std. Dev.	0.1228	0.2486	0.3277	0.0881	0.0071	0.0027	10.1547	2.4045
Percentage			Quantiles					
1	0.2242	0.1578	-1.4733	0.2064	0.0014	0.0003	7.9354	0.3378
5	0.2919	0.2638	-1.0663	0.2991	0.0023	0.0008	11.5932	0.4218
10	0.3020	0.3328	-0.8300	0.3572	0.0028	0.0014	13.6045	0.4857
25	0.3337	0.4332	-0.5623	0.4441	0.0044	0.0028	17.0002	1.1010
50	0.3979	0.5898	-0.3296	0.5017	0.0081	0.0048	23.1412	2.4978
75	0.4699	0.8225	-0.2025	0.5325	0.0132	0.0063	29.7766	3.6711
90	0.6700	0.9764	-0.0396	0.5660	0.0190	0.0084	40.6698	4.8467
95	0.7047	1.0339	0.0177	0.5763	0.0251	0.0104	44.9378	7.7727
99	0.7429	1.1989	0.0876	0.6594	0.0334	0.0121	50.1391	12.8423

	Goodness of Fit Metrics			
	rmse	aae	ape	number of options
Mean	0.0738	0.0562	0.0316	35.8523
Std. Dev.	0.0683	0.0481	0.0171	10.2221

TABLE 2

Calibration Results on GS of Credit and Liquidity Modified VGSSD Sato process

	Option Surface Parameters				Liquidity Parameters		Credit Parameters	
	sigma	nu	theta	gamma	lambda	eta	c	a
Mean	0.3527	0.6832	-0.3524	0.4696	0.0056	0.0079	34.8995	4.0117
Std. Dev.	0.0388	0.2632	0.1493	0.0578	0.0058	0.0037	15.2737	3.2182
Percentage			Quantiles					
1	0.2736	0.2994	-0.7598	0.3448	0.0000	0.0011	16.7497	0.6302
5	0.3059	0.3876	-0.6798	0.3667	0.0001	0.0016	18.6538	0.6523
10	0.3153	0.4018	-0.5646	0.3826	0.0002	0.0020	19.8705	0.7349
25	0.3300	0.5329	-0.4183	0.4280	0.0006	0.0055	23.7206	2.0825
50	0.3439	0.6120	-0.3335	0.4832	0.0027	0.0080	31.9683	3.3678
75	0.3751	0.7972	-0.2680	0.5195	0.0097	0.0107	41.4322	4.9972
90	0.3865	1.0111	-0.2148	0.5307	0.0155	0.0125	53.3556	7.6455
95	0.4163	1.1154	-0.1022	0.5489	0.0163	0.0137	70.9174	11.0731
99	0.5001	1.8070	-0.0013	0.5680	0.0176	0.0145	84.5815	16.0059

			Goodness of Fit Metrics	
	rmse	aae	ape	number of options
Mean	0.5957	0.3662	0.0329	77.2869
Std. Dev.	0.9271	0.4595	0.0352	16.5025

TABLE 3

Calibration Results on JPM of Credit and Liquidity Modified VGSSD Sato process

	Option Surface Parameters				Liquidity Parameters		Credit Parameters	
	sigma	nu	theta	gamma	lambda	eta	c	a
Mean	0.3647	0.6998	-0.4213	0.4797	0.0052	0.0102	19.5679	3.0458
Std. Dev.	0.0768	0.1536	0.1733	0.0507	0.0046	0.0055	9.1157	1.6106
Percentage			Quantiles					
1	0.2261	0.3615	-0.9429	0.3818	0.0000	0.0033	11.7712	0.7222
5	0.2658	0.4466	-0.7348	0.3914	0.0003	0.0044	12.3632	0.7436
10	0.2905	0.4705	-0.6885	0.4020	0.0003	0.0051	13.1412	0.9873
25	0.3198	0.5889	-0.5387	0.4426	0.0009	0.0065	14.4034	1.7811
50	0.3486	0.7141	-0.3557	0.4914	0.0038	0.0100	15.7181	2.8846
75	0.3977	0.8099	-0.2987	0.5198	0.0088	0.0109	19.5962	4.1092
90	0.4488	0.8939	-0.2573	0.5379	0.0125	0.0161	37.4757	4.8703
95	0.5592	0.9530	-0.2290	0.5537	0.0137	0.0208	44.1697	5.9696
99	0.6000	0.9931	-0.2013	0.5690	0.0164	0.0319	47.4503	7.8637

	Goodness of Fit Metrics			
	rmse	aae	ape	number of options
Mean	0.0948	0.0705	0.0290	58.0802
Std. Dev.	0.0496	0.0335	0.0128	16.0066

TABLE 4

Calibration Results on WFC of Credit and Liquidity Modified VGSSD Sato process

	Option Surface Parameters				Liquidity Parameters		Credit Parameters	
	sigma	nu	theta	gamma	lambda	eta	c	a
Mean	0.3525	0.7594	-0.3678	0.4600	0.0104	0.0153	26.1688	2.6124
Std. Dev.	0.1044	0.2358	0.4022	0.0674	0.0071	0.0090	15.2591	1.3752
Percentage			Quantiles					
1	0.1583	0.2668	-1.0809	0.3429	0.0000	0.0041	11.2826	0.5197
5	0.1961	0.3156	-0.9423	0.3520	0.0013	0.0052	11.4094	0.5487
10	0.2161	0.3950	-0.8554	0.3609	0.0019	0.0069	11.8162	0.6205
25	0.3090	0.6292	-0.5525	0.4064	0.0052	0.0094	13.5901	1.5983
50	0.3442	0.7750	-0.2927	0.4576	0.0094	0.0119	21.3955	2.8544
75	0.3790	0.9263	-0.2671	0.5209	0.0134	0.0207	31.8577	3.6566
90	0.4731	1.0302	-0.1614	0.5462	0.0217	0.0267	51.3587	4.1181
95	0.6050	1.1378	0.3152	0.5535	0.0249	0.0319	58.2857	4.3507
99	0.6801	1.2210	1.2665	0.5575	0.0288	0.0478	65.9812	5.8069

			Goodness of Fit Metrics	
	rmse	aae	ape	number of options
Mean	0.1002	0.0571	0.0349	43.7173
Std. Dev.	0.0785	0.0543	0.0161	10.5690