Pricing to Acceptability: With applications to valuing one's own credit risk.

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Abstract

The theory of pricing to acceptability developed for incomplete markets is applied to marking ones own default risk. It is observed in agreement with Heckman (2004), that assets and liabilities are not to be valued in financial reporting at the same magnitude. Liabilities are marked at ask prices that are above the asset mark at bid prices. Applying cones of acceptability defined by concave distortions it is observed that counterintuitive profitability resulting from credit deterioration is mitigated. We argue that the difference between the liability mark at ask and the asset mark at bid be taken as an upfront expense deposited in a special account called the $ODOR$ account for Own Default Operating Reserve. Procedures are described for pricing coupon bonds separately as assets and liabilities. These procedures employ the default time distribution embedded in the CDS market.

1 Introduction

A perturbing development now frequently reported in the press is the adverse effect on revenues of improvements in credit conditions. Equivalently CEO's explaining earnings statements comment on the profits being reported as a consequence of a deterioration in ones own credit rating. This situation is a consequence of valuing ones own credit risk in reporting liabilities. A reduced liability is reported when one is less likely to make promised payments and hence the resulting profit. The likelihood is under the risk neutral probability and in this regard we note the findings of Collin-Dufresne, Goldstein and Martin (2001) that question the linkage between credit spreads and default probabilities.

By way of examples we quote from Risk magazine, October 22 2009 as follows. "Morgan Stanley announced a \$757 million profit for the third quarter on October 21, compared with a loss of \$159 million in the second quarter. However, it said it had taken a \$0.9 billion loss as CDS spreads referencing the bank's own debt narrowed further. Five year senior CDS spreads on the firm moved from 191 basis points to 138bp across the quarter. ... However in 2008, the bank's fixed-income business alone enjoyed a profit of \$3.5 billion on its own liabilities due to the bank's credit spreads ballooning from 98bp to 402bp over the course of the year."

Philip Heckman (2004) argued against this practice, pointing to the above counterintuitive consequences of making such credit adjustments. He advocated instead that the liability should always be reported on a going concern basis and be valued as a default free liability. However, he agreed that the lower value obtained by discounting at the higher credit sensitive risky rate was admissible as an asset value. In particular, he argued that the same cash flow viewed as an asset should be valued differently when it is a liability.

Wallace (2004) offers a rebuttal of Heckman pointing out that modern financial theory recognizes the need for higher returns related to greater risks and two equivalent promises from counterparties with differing capacity to pay are not the same product and hence do not have the same value. However, their value though different on account of the capacity to pay, is the same for each promise, on both sides of the balance sheet by virtue of the law of one price in markets.

We argue that the problem arises in markets with fading liquidity characterized by substantial inter transaction times. In such markets, the law of one price fails and the terms of trade begin to depend on the trade direction. Specifically it now matters whether one is buying from or selling to the market. Transaction prices differ significantly with the trade direction for parties treating the illiquid market as a counterparty. Standard theory, abstracting from market liquidity considerations, may therefore seriously misguide decisions and valuations. The reasons for illiquidity and the resulting spreads in markets we focus on are more closely related to the consequences of market completeness, (Zurita (2008)). There is then a necessity for the market as a counterparty to hold residual or unhedgeable risks. This need translates into bid and ask spreads as studied in Bernardo and Ledoit (2000), Cochrane and Saa-Requejo (2000), Černý and Hodges (2000), Carr, Geman and Madan (2001) and Jaschke and Kuchler (2001).

There is a well developed financial theory for the pricing and valuation of liquid assets where the inter-transaction time is often below a minute. Here the law of one price prevails, bid ask spreads are zero, and all claims are priced as discounted expectations under an equilibrium pricing measure or probability. However, the market for individual specific credit sensitive debt is too specific to be liquid. Though there may be a liquid market for one's debt where it is traded as an asset in the portfolios of various other counterparties, this is not the market for one's own debt.

When some other market participant sells one's outstanding debt, they do not issue a new security with a simultaneous commitment to make coupon and face value payments. They merely transfer the receipts for such from themselves to others. They are trading an existing stream of payments and not entering a contract to create a new stream of cash flows. The sale and purchase of one's debt between other market participants has little to do with one's own ability to make these payments. They may reflect a market assessment of such an ability but do not directly impact this ability.

The price of one's own liability on the other hand may be related either to the price at which a new issue will trade at the margin or the price at which one may buy back existing issues. These prices can be substantially disconnected from the price at which old or existing issues transact among others. An increase in the outstanding debt may be coupled with concerns about one's ability to carry the additional debt burden, while a repurchase may benefit from a reduction of the debt burden leading to a higher price for the repurchase and a lower price for the additional sale.

In general such considerations reflect the fact that one sells into illiquid markets for a bid price that is lower than the ask price at which one buys back from such markets. The market for one's own debt is illiquid by virtue of substantial intertransaction times. There is therefore no market price for one's own debt, as these transactions occur infrequently. However, we need to recognize that the price at which we buy back will be substantially higher than what we get for selling additional debt to the market and both these prices differ from how our debt trades for among others in the market. The relevant price for a liability is the price at which we may buy back the debt and this is the ask price of what is an illiquid market.

Financial reporting in incomplete markets has to recognize the consequences of such illiquidity embedded in the presence of bid ask spreads. As a consequence, assets when they are sold, go out at the bid price of counterparties and liabilities when reversed, must pay the ask price. What is missing in standard theory and addressed in existing theories on market incompleteness, is that assets and liabilities are not to be equally valued. We apply extant theories for bid and ask prices in incomplete markets to gain further insights on these valuation questions. A recent contribution by Easley and O' Hara (2010) studies these questions in the context of a no trade equilibrium model with pricing formulas for bid and ask prices that are similar, at a mathematical level, to the ones developed here.

The primary theoretical insight of two price markets is the recognition upfront of the terms at which markets are comfortable with holding residual risk. Incompleteness means that this risk is present and cannot be eliminated. Exact replication using liquid assets is out of the question and the best hedge leaves one exposed to residual risk. The question to be answered is what residual risk will be willingly held by market participants. One answer is obvious, any residual risk that is always positive to a party, will be willingly held by that party. These considerations lead to ask prices being the lowest cost of superreplication while bid prices are the highest cost of subreplication. The resulting bid ask spreads are however huge and not relevant for practical considerations.

It is clear that most market participants will take some exposure to loss and cannot insist that the residual is always positive for them. Artzner, Delbaen, Eber and Heath (1999) axiomatized acceptable risks as some convex cone containing the cone of nonnegative cash flows. Based on this formulation for acceptable risks Carr, Geman and Madan (2001) reformulated the problem for constructing bid and ask prices and showed as expected that the spread comes down as we widen the cone of acceptable risks. But there was no effective description of the cone and no way to compute either the bid or the ask price.

Further progress was made in the literature on good deal bounds and we cite Bernardo and Ledoit (2000), Cochrane and Saa-Requejo (2000), Černý and Hodges (2000), Jaschke and Küchler (2001) and Staum (2008). However as Staum (2008) observes, "there is as yet no fully developed, sound theoretical framework for pricing derivative securities in incomplete markets." Staum (2008) further states, "we want a methodology that respects the no-arbitrage bounds, is computationally efficient,. . . ."

Recently a family of computationally efficient cones of acceptable risks were introduced in Cherny and Madan (2009). In the context of a static model with a fixed horizon, Cherny and Madan (2009) relate cones of acceptability that depend solely on the probability law of a cash flow, to positive expectations under concave distortions. For such probability law dependent cones, the decision on whether a risk is acceptable or not is by construction determined by its probability distribution. Hence in deciding on the acceptability of a random cash flow X, all we need to know is its probability distribution $F(x) = \Pr(X \leq x)$. Such cones may be constructed from a concave distribution function $\Psi(u)$ defined on the unit interval, used as a distortion of probability, by the condition that X is acceptable just if

$$\int_{-\infty}^{\infty} x d\Psi(F(x)) \geq 0. \tag{1}$$

Cherny and Madan (2009) go on to construct a parameterized sequence of cones $\Psi^{\gamma}(u)$ associated with a decreasing sequence of cones $\mathcal{A}_{\gamma}$ that start at a half space for $\gamma = 0$ and finish at the non-negative cash flows as γ tends to infinity. Such cones of acceptability may be broadly viewed as protecting against model uncertainty as envisaged in Rigotti and Shannon (2005) building on the earlier work of Gilboa and Schmeidler (1989).

This theory of pricing to acceptability using convex cones has now been employed to compute bid and ask prices in Eberlein and Madan (2009), Madan (2009), and closed forms have now been derived for options in Cherny and Madan (2010). In most applications the distortion used is termed *minmaxvar* where the parameter γ represents the stress level that increases with γ and for *minmaxvar*,

$$\Psi^{\gamma}(u) = 1 - (1 - u^{\frac{1}{1+\gamma}})^{1+\gamma}. \tag{2}$$

To shed some light on the issues of pricing ones own credit risk we shall follow Heckman (2004) and Wallace (2004) and limit the discussion to the simplest debt instrument, a 10 year zero coupon, non callable, and not putable bond. The face value is 10000 and it is issued by two companies here denoted U, V with risky discount rates of $7, 12$ percent and present values of 5083, 3220 with risk free discount at 5.8% and default free price of 5690. We shall first consider the pricing of assets and liabilities in the absence of hedging assets. We shall briefly comment later on the impact of hedging assets but leave the further pursuit of these details to applications of the results obtained in Cherny and Madan (2010).

The procedures and principles advocated in this paper address the immediate high priority tasks formulated at the Summit on Financial Markets and the World Economy in Washington November 15, 2008. In times of stress illiquidity is enhanced and we advocate that complex instruments and swaps be partitioned into their positive and negative parts with the former being priced as an asset on the left of the balance sheet and the latter being priced as a liability on the right hand side of the balance sheet. There should be no off-balance sheet items whatsoever. Furthermore we detail specific procedures for pricing assets and liabilities and thereby begin to address quantitative financial rules for balance sheets calibrated to observable and measurable stress levels in markets. We agree with Ryan (2008) that the recent crisis is a teachable moment that should be used to refine our methodologies and here we present some solutions for derivatives in this regard.

We find that the advocacy of Heckman was surprisingly accurate when viewed from our incomplete markets perspective. Our procedures place the value of our own debt closer to the prices for default free bonds and makes the spread between Treasury bonds and corporate bonds relevant to the spread between bid and ask prices for corporate liabilities. The studies of Chen, Lesmond, Wei (2007), Chen (2007), Cremers, Driessen, Maenhout and Weinbaum (2008), Tauchen and Zhou (2010) are thereby made relevant to the valuation of corporate liabilities. We recognize that such spreads could be reduced by our own methodology provided one effectively implemented hedges for one's own default risk. However, we leave for future research an analysis endogenizing such hedges possibly along the lines of Gatev and Strahan (2006) or by trading index credit default swaps to hedge systematic components as described in Bielecki, Jeanblanc and Rutkowski (2008).

The outline of the rest of the paper is as follows. Section 2 reviews the essentials of acceptability pricing as set out in Artzner, Delbaen, Eber and Heath (1999) and its subsequent development, relating these concepts to the financial valuation issues being considered here. Section 3 takes up the example studied in Heckman (2004) and Wallace (2004). Section 4 applies these methods to redo the financial statements of the major banks through the recent crisis with a view to correcting the counterintuitive remarks we opened with by reversing the profitability of credit deterioration. Section 5 extends the computations from pure discount bonds to coupon bonds. Section 6 comments on the broad relationships between asset and liability pricing, noting in particular that the negative part of an asset is a liability and the mechanism of pricing it as an asset automatically prices the negative part as a liability. Section 7 describes a potential calibration strategy for a stress level appropriate for liability pricing. For this purpose we employ the CDS market to extract relevant risk neutral probabilities, by treating this CDS market as a liquid market for our promises traded as an asset, by numerous counterparties. Section 8 comments on the effects of hedging assets. Section 9 concludes.

2 Essentials of Pricing to Acceptability

This section introduces the general principles of pricing to acceptable levels of risk. Specific formulas for such pricing using concave distortions are then developed. We close with a comment on the choice of the base measure used in defining acceptable risks.

2.1 General Principles of Acceptability Pricing

We have observed that the set of risks viewed as random variables X on a probability space $(\Omega, \mathcal{F}, P)$ that are acceptable to the general economy are modeled as a cone containing the nonnegative cash flows, as the latter are always acceptable by virtue of being devoid of risk. For purposes of corporate reporting, one is in principle describing the state of corporate affairs to the external world. The corporate reports are not just a statement evaluating shareholder wealth, but they report as profit what the external world must agree as legitimately withdrawable funds from the enterprise. One therefore has to model the risks the external world is willing to accept.

In the complete markets context of modern finance theory and its liquid assets, this profit is clear and equals the market value of the cash flow accessed. Under the unique 'pricing' or so-called 'risk neutral' measure Q one merely evaluates

$$E^Q\left[X\right], \tag{3}$$

for a zero cost cash flow. If this expectation is positive one undertakes the activity generating X as we then have a positive α adequately compensating risks.

When markets are incomplete, the cone generated by a single measure is too wide. Acceptable risks in Artzner, Delbaen, Eber and Heath (1999) are defined by a convex set of supporting probability measures $Q \in \mathcal{M}$ with the property that X is acceptable just if

$$E^Q[X] \geq 0, \text{ for all } Q \in \mathcal{M}, \text{ or} \tag{4}$$

$$\inf_{Q \in \mathcal{M}} E^Q[X] \geq 0. \tag{5}$$

The class of externally acceptable cash flows is then considerably smaller than the positive α cash flows, and the acceptability requirement is considerably more conservative.

When we come to marking assets and liabilities with nonnegative cash flows to be received or paid out we have to ask what are the externally acceptable terms. For an asset with a random cash flow $\widetilde{A} \geq 0$ or a liability with a random cash flow $\widetilde{L}$ the prices A, L that are externally acceptable are such that the residuals

$$\widetilde{A} - A, \; L - \widetilde{L} \tag{6}$$

are externally acceptable.

It follows immediately that the smallest admissible value for L and the largest admissible value for A is given by

$$L = \sup_{Q\in\mathcal{M}} E^Q\left[\widetilde{L}\right] \tag{7}$$

$$A = \inf_{Q\in\mathcal{M}} E^Q\left[\widetilde{A}\right]. \tag{8}$$

For the same cash flow its value as an asset is then lower than its value as liability. Consistent with the views of Heckman, asset valuation and the valuation of liabilities are different but related activities. In particular, the cash flow is always worth more as a liability than as an asset. This is none other than the recognition that assets must be sold at bid prices while liabilities are unwound at the ask price.

2.2 Acceptability Pricing Using Distortions

The question that now arises is, "How do we compute these bid and ask prices or equivalently the marks on the asset and liability side of the balance sheet?". For this we turn to Cherny and Madan (2009). Suppose first that acceptability is to be defined completely by the probability law or distribution function $F(x)$ of the risk at hand. Cherny and Madan (2009) then describe the link between acceptability and concave distortions of the distribution function as expressed in equation (1) for some concave distortion Ψ. The set of supporting measures $\mathcal{M}$ for this set of acceptable risks is all measures Q with density $Z = \frac{dQ}{dP}$ satisfying the condition

$$E^P\left[(Z-a)^+\right] \leq \Phi(a) = \sup_{u\in[0,1]} \left(\Psi(u) - ua\right), \text{ for all } a \geq 0 \tag{9}$$

In summary, the condition (1) defines a valid cone of acceptable risks that depend on just a knowledge of the distribution function of the cash flow. We may observe on rewriting the integral in condition (1), assuming that F has a density f, as

$$\int_{-\infty}^{\infty} x\Psi'(F(x))f(x)dx, \tag{10}$$

that our expectation under concave distortion is also an expectation under a measure change. We note that large losses with $F(x)$ near zero are reweighted upwards by $\Psi'(F(x))$ as Ψ' decreases for any concave distortion. The more concave the distortion the higher the upward reweighting of losses and the more difficult it is to be acceptable.

Cherny and Madan (2009) go on to propose a sequence of concave distortions indexed by a real number γ that are increasingly more concave with a corresponding decreasing sequence of sets of acceptability. The recommended distortion that we employ in this paper is *minmaxvar* with Ψ^γ as defined in (2)

A simple computation yields the equation for the asset value, for the cone indexed by γ, as

$$A = \int_{-\infty}^{\infty} x d\Psi^\gamma(F_{\widetilde{A}}(x)) \tag{11}$$

with a computation associated with a simulated set of cash flows sorted into increasing order as $x_1 \leq x_2 \leq \cdots x_N$ by

$$C \approx \sum_{j=1}^{N} x_j \left(\Psi^\gamma \left(\frac{j}{N} \right) - \Psi^\gamma \left(\frac{j-1}{N} \right) \right). \tag{12}$$

For the liability value the comparable result yields

$$L = -\int_{-\infty}^{\infty} x d\Psi^\gamma(F_{-\widetilde{L}}(x)). \tag{13}$$

In this case we sort into increasing order a simulation from the law of $-\widetilde{L}$, $x_1 \leq x_2 \leq \cdots x_N$ and evaluate

$$L = -\sum_{j=1}^{N} x_j \left(\Psi^\gamma \left(\frac{j}{N} \right) - \Psi^\gamma \left(\frac{j-1}{N} \right) \right). \tag{14}$$

One may show directly from the concavity of Ψ^γ that $L \geq A$ for the same random variable (Madan (2009)).

2.3 Remarks on the base measure

The earlier papers defining acceptability, for example Artzner, Delbaen, Eber and Heath (1999) and its follow up papers up to Cherny and Madan (2009) took the base measure to be the physical or true measure P. We note now that from the perspective of reporting the state of affairs of an entity to the external or general economy this may not be an adequate base measure. This is because a positive expectation that fails to earn sufficient compensation for risk undertaken may not be approved by the external economy. From the perspective of reporting to the external economy we may wish to take as a base measure a particular risk neutral measure with a positive expectation now being a positive alpha trade. Approving all positive alpha trades may be too generous given market incompleteness and hence we may reduce the cone of acceptability by requiring positive expectation with respect to a convex set of measures equivalent to our base risk neutral measure. For notational convenience we still refer to this base measure as the P measure, noting now that it could be taken to be risk neutral.

In summary, effective pricing to acceptability may be accomplished on first selecting a base risk neutral measure. This measure should be viewed as explicitly or implicitly defining societal trading norms in markets or regulatory interventions when they apply. We then work out cash flow distributions under this measure. Bid prices are then straightforwardly evaluated by distorted expectations while ask prices are the negative of the bid price for the negative cash flow.

3 The Heckman example using acceptability pricing

This section applies the principles of the previous section to pricing a pure discount bond studied in the original Heckman (2004) paper. We then explain how pricing to acceptability supports analytically the Heckman advocacy.

3.1 The Heckman example

What is the value of the liability and what is the asset value for promises of companies U, V in the Heckman examples? In present value terms we ask what amount of money L received for the promised payout and invested at the risk free rate makes the residual cash flow acceptable to the external world. In this case the residual cash flow is

$$L(1.058)^{10} - 10000 \times \mathbf{1}_S \tag{15}$$

where S is the survival or non default set. The answer as we have seen, now including time value considerations is

$$L = \frac{1}{(1.058)^{10}} 10000 \sup_{Q \in \mathcal{M}} E^Q[\mathbf{1}_S]. \tag{16}$$

Similarly the value as an asset is

$$A = \frac{1}{(1.058)^{10}} 10000 \inf_{Q \in \mathcal{M}} E^Q \left[\mathbf{1}_S\right]. \tag{17}$$

If we agree that the asset prices are as in the Heckman example, at 5083, and 3220 for U, V respectively then as a liability it is higher.

If we use the *minmaxvar* cone at level γ then as the probability distribution here is a simple one with default probabilities p_U, p_V respectively for U, V we get that

$$5083 = \frac{1}{(1.058)^{10}} 10000 \times \left(1 - \Psi^{\gamma}(p_U)\right), \tag{18}$$

$$3220 = \frac{1}{(1.058)^{10}} 10000 \times \left(1 - \Psi^{\gamma}(p_V)\right), \tag{19}$$

and the liability values are respectively

$$L_U = \frac{1}{(1.058)^{10}} 10000 \times \Psi^{\gamma}(1 - p_U), \tag{20}$$

$$L_V = \frac{1}{(1.058)^{10}} 10000 \times \Psi^{\gamma}(1 - p_V). \tag{21}$$

For a start we use *minmaxvar* at level 0.75 as advocated in Madan (2009b). Other levels are considered later in the paper. For this stress level, we have

$$p_U = 0.0078 \tag{22}$$

$$p_V = 0.1062 \tag{23}$$

$$L_U = 5689 \tag{24}$$

$$L_V = 5643 \tag{25}$$

For $\gamma = .5$ the results are

$$p_U = 0.0195 \tag{26}$$

$$p_V = 0.1775 \tag{27}$$

$$L_U = 5682 \tag{28}$$

$$L_V = 5447 \tag{29}$$

One may graph the value of the asset and liability as a function of the default probability. We present in Figure 1 the graph of both the asset value and the liability value for default probabilities ranging from 0.0010 to 0.2. We observe that the asset value is very sensitive to this probability with the price dropping from around 5500 to around 2500. If the liability were marked the same way a credit deterioration would reflect an enormous profitability associated with this deterioration. When marked to acceptability however the value of the liability barely moves from the risk free level of 5690 down to 5600. The result is very close to the Heckman advocacy of just using the risk free value of 5690.

3.2 Analytical support for the Heckman advocacy

There is analytical support for the Heckman advocacy in pricing to acceptability as the liability valuation is

$$L = \frac{1}{(1.058)^{10}} 10000 \times \Psi^{\gamma}(1-p) \tag{30}$$

Now we may write as an approximation valid for small default probabilities p

$$\Psi^{\gamma}(1-p) \approx 1 - \Psi^{\gamma\prime}(1)p + \frac{1}{2}\Psi^{\gamma\prime\prime}(1)p^2 \tag{31}$$

We have advocated the use of distortions that discount large gains down to zero and this requires that $\Psi^{\gamma\prime}(1) = 0$ and hence to first order in probability $\Psi^{\gamma}(1-p) \approx 1$ or we use the risk free rate for discounting as advocated by Heckman.

In the case of *minmaxvar* we have

$$\Psi^{\gamma\prime\prime}(x) = -\frac{\gamma}{1+\gamma}\left[\left(1 - x^{\frac{1}{1+\gamma}}\right)^{\gamma-1} x^{-\frac{2\gamma}{1+\gamma}} + \left(1 - x^{\frac{1}{1+\gamma}}\right)^{\gamma} x^{-\frac{2\gamma+1}{1+\gamma}}\right] \tag{32}$$

and we see that if we take $\gamma > 1$ then $\Psi^{\gamma\prime\prime}(1)$ is also zero. For high values of γ the Heckman prescription is absolutely correct. There is some minimal leniency in relaxing the liability value for wider and more tolerant cones for acceptable risk and when default probabilities are high. The sensitivity of the liability value to the default probability is however substantially reduced.

In summary we observe that pricing a liability like a pure discount bond to its ask price computed with a view to making the residual acceptable will lead to valuations that are relatively stable and less sensitive to variations in the credit spread. The effects on profits arising from credit deterioration are thereby minimized.

4 PnL impact of credit changes

This section tracks the effect of changes in credit spreads on profits when we follow the law of one price and price liabilities as if they were assets and contrast these values with liabilities priced at the ask price of an incomplete market. We also report on how to manage the difference in the balance sheet through a reserve account that we call the own default operating reserve ($ODOR$) account.

4.1 Credit spreads and profits

We consider a 5 year maturity with a face value of 35 billion dollars and an interest rate of 2%. We take credit spreads at quarter ends of 301, 402, 296, 191, and 138 basis points. We price the debt as an asset and as a liability using $minmaxvar$ at stress level 0.75 and report the PnL impact across four successive quarters in Table 1. A wider range of stress level is considered later in Table 8 below.

A graph of the PnL impact is presented in Figure (2). The asset and liability values are presented at the end of each quarter in Table 2.

4.2 The ODOR account

The question now arises as to how one should account for the difference between the mark as a liability and the mark as an asset. If a lender of last resort in fact existed one could price the put option and show it as an asset purchased for the expense given by the difference in values. Of course this is not the case, though we agree with Heckman that the difference is an expense but where does it go. A possible location is as a cash reserve against ones own default. This can be our $ODOR$ account. In our example as the credit situation changes the $ODOR$ reserve varies reflecting the value of the reserve appropriate for own default in line with the current default probability. Table 3 gives the levels of the $ODOR$ account at quarter end. The level of this account represents the level of deterioration in ones own credit quality.

In summary, pricing liabilities to incomplete market ask prices significantly reduces the variations due to changes in credit spreads. Worsening spreads do not lead to profits but just lead to increases in the $ODOR$ account that cannot be and should not be distributed as a realized profit.

5 Bid and Ask Prices for coupon bonds

This section develops the general formulas for pricing coupon bonds as assets and as liabilities. The inputs for the computations are the probability distribution of the default time, the particular distortion chosen and its stress level.

Consider now a sequence of payments in the amount c_i to be made or received at time t_i for a finite sequence of times $t_1 < t_2 < \cdots < t_n$. Let the random default time be τ with distribution function

$$P(\tau \leq t) = F(t), \ t \geq 0. \tag{33}$$

Let us first consider two payments or receipts c_1, c_2 at times $t_1 < t_2$. First we model the bid price as an asset. Let the risk free discount factors be d_1 and d_2. There are three possible present value payments and they are 0, c_1d_1, $c_1d_1 + c_2d_2$. The probabilities are respectively $F(t_1)$, $F(t_2) - F(t_1)$, and $1 - F(t_2)$. The distribution function at the three cash flows is $F(t_1)$, $F(t_2)$, 1. The bid price or price as an asset, A, is

$$A = c_1d_1 \left(\Psi^{\gamma}\left(F(t_2)\right) - \Psi^{\gamma}(F(t_1))\right) + (c_1d_1 + c_2d_2)\left(1 - \Psi^{\gamma}\left(F\left(t_2\right)\right)\right). \tag{34}$$

As a liability the cash flows in increasing order are $-(c_1d_1 + c_2d_2)$, $-c_1d_1$, and 0. The probabilities are now $1 - F(t_2)$, $F(t_2) - F(t_1)$, and $F(t_1)$. The distribution function for the negative of the cash flow is

$1 - F(t_2), 1 - F(t_1)$, and 1. The ask price or price as a liability, L,

$$L = (c_1 d_1 + c_2 d_2)\Psi^\gamma\left(1 - F(t_2)\right) + c_1 d_1\left(\Psi^\gamma\left(1 - F(t_1)\right) - \Psi^\gamma\left(1 - F(t_2)\right)\right). \tag{35}$$

More generally for n payments we have

$$A = \sum_{j=1}^{n}\left(\sum_{i=1}^{j} c_i d_i\right)\left[\Psi^\gamma\left(F(t_{j+1})\right) - \Psi^\gamma\left(F(t_j)\right)\right] \tag{36}$$

where $t_{n+1} = \infty$. For a liability we have

$$L = \sum_{j=1}^{n}\left(\sum_{i=1}^{j} c_i d_i\right)\left[\Psi^\gamma\left(1 - F(t_j)\right) - \Psi^\gamma\left(1 - F(t_{j+1})\right)\right]. \tag{37}$$

For a specific example let us take a default time distribution in the Weibull family with

$$F(t) = 1 - \exp\left(-\left(\frac{t}{c}\right)^a\right),\ t \geq 0 \tag{38}$$

with five annual payments of 1000, 2000, 500, 700, and 4000 dollars. Let the risk free rates be .01, .0125, .015, .02, .025. The discount curve is .99, .9753, .9560, .9231, and .8825. The price as a risk free set of cash flows is 7594.84.

For three settings for c of 10, 15, and 20 and three settings for a of 1.1, 1.5, and 2.0 we compute using the expressions (36) and (37) the price for this claim as an asset and a liability. The distortion used is again *minmaxvar* at the level 0.75. The results are presented in Table 4 and Table 5 respectively for an asset and a liability.

6 General remarks relating bid and ask prices

This section comments on bid ask prices for real valued cash flows, showing in particular that the bid price of an asset with negative component, automatically values the negative part as if it was a liability. We next present closed forms for bid and ask prices for a fair value swap with a Gaussian distribution. We close with a comment on aggregation considerations.

6.1 Bid and Ask decompositions of swaps

The random cash flow X to be priced as an asset or a liability could have both positive and negative components. We may also write the cash flow as the difference of its positive and negative components as

$$X = X^+ - X^- \tag{39}$$

where X^+ is an asset and X^- is a liability. We observe in this section that the bid price of X is the bid price of X^+ less the ask price of X^-. Hence X^- is treated like a liability and priced at the ask and X^+ is an asset priced at the bid.

The bid price for X is by construction

$$b(X) = \int_{-\infty}^{\infty} x d\Psi \left(F_X(x) \right) \tag{40a}$$

while the ask price is

$$a(X) = -\int_{-\infty}^{\infty} x d\Psi \left(F_{(-X)}(x) \right) \tag{41}$$

Consider now the bid price of $(-X)$ and this is

$$\begin{aligned} b((-X)) &= \int_{-\infty}^{\infty} x d\Psi\left(F_{(-X)}(x)\right) && (42) \\ &= -a(X) && (43) \end{aligned}$$

Now the bid price of X^+ is

$$b(X^+) = \int_0^{\infty} x d\Psi\left(F_X(x)\right) \tag{44}$$

also the ask price of X^- is

$$\begin{aligned} a(X^-) &= -\int_{-\infty}^{0} x d\Psi\left(F_{(-X^-)}(x)\right) && (45) \\ &= -\int_{-\infty}^{0} x d\Psi\left(F_X(x)\right) && (46) \end{aligned}$$

Hence

$$b(X) = b(X^+) - a(X^-). \tag{47}$$

Similary by applying (42) we learn that

$$\begin{aligned} a(X) &= -b((-X)) && (48) \\ &= a\left((-X)^-\right) - b\left((-X)^+\right) && (49) \\ &= a(X^+) - b(X^-) && (50) \end{aligned}$$

so when X is a liability its negative part is an asset and it is priced at bid while its positive part is a liability and this is priced at the ask.

In general for swap type contracts we recommend that they be split into their positive and negative parts with the positive part being priced at bid and the negative part priced at ask. The situation arises here of a zero net value swap traded at market that is immediately marked negatively as the bid price reduces the value of the positive part and the ask price raises the value of the negative part. The difference is however not a loss but should be taken as a reserve against possibly disadvantageous future unwinds.

6.2 Gaussian fair value swaps

Consider a fair value swap with a normally distributed cash flow X with a zero risk neutral mean and a volatility of σ. We may partition the swap into its positive and negative parts and price each at its bid and ask price. We have that

$$F_{X^+}(x) = N\left(\frac{x}{\sigma}\right),\ x \geq 0, \tag{51}$$

$$F_{X^-}(x) = 1 - N\left(\frac{-x}{\sigma}\right),\ x \geq 0. \tag{52}$$

The bid price of X^+ is

$$b(X^+) = \int_0^\infty x d\Psi\left(N\left(\frac{x}{\sigma}\right)\right) \tag{53a}$$

$$= \sigma \int_0^\infty y d\Psi\left(N(y)\right). \tag{53b}$$

The ask price of X^- on the other hand is

$$a(X^-) = -\int_{-\infty}^0 x d\Psi\left(N\left(\frac{x}{\sigma}\right)\right) \tag{54}$$

$$= -\sigma \int_{-\infty}^0 y d\Psi(N(y)). \tag{55}$$

Computing these integrals numerically for the distortion *minmaxvar* at the stress levels .25, .5, and .75 we have the bid prices at $.2251\sigma$, $.1248\sigma$ and $.0679\sigma$ while the corresponding ask prices are $.6408\sigma$, $.9045\sigma$ and 1.1755σ respectively. The reserve on a million dollar notional with a 10% volatility and stress level .5 is around $78,000$ dollars. This goes down to $41,573$ dollars if the stress level is reduced to .25. Eberlein and Madan (2010) argue that for a Gaussian model the appropriate stress level is .2222.

6.3 Aggregation considerations

From the structure of bid pricing seen as an infimum of admissible valuations it is clear that the bid price of a package is greater than the sum of the bid prices of the components. Similarly the ask price of a package is lower than the sum of the ask prices of components. Hence one should aggregate claims into packages and perform bid ask valuations at the level of collections of assets sharing common risk exposures.

In summary we observe that for swaps seen as assets their negative parts are liabilities and should be marked as such on the liability side, while the positive part is priced at bid as an asset. The difference would go to reserves and would help minimize short term profit taking. Of course we recognize that this gap can be reduced by packaging into suitable aggregates and this should be permitted.

7 Liability Pricing via Stress Calibration

This section illustrates how one may employ the *CDS* market to extract the default time distribution. We then show how one uses the stress levels implicit in the price of one's liability as an asset for others, to price the liability at its ask price. We also perform such a calibrated liability pricing for a sample of banks.

7.1 Weibull default times and the CDS market

We take the position that the market for our own debt as an asset for others may have a relatively liquid market as there are many counterparties for our own debt who could trade directly with us or between themselves. We may take for the bid price some transaction prices in the lower end of observed transactions. Furthermore from the CDS market on our own name we may estimate a risk neutral distribution for the default time in the Weibull family to estimate the parameters c, a. One may then employ equations (36) and (37) to construct the asset and the liability price as a function of the stress level.

A graph for our example in section 5 with $c = 10$, $a = 1.25$ is shown in Figure 3. Given the bid or asset price we may read off the stress level along the blue curve and then get the liability price from the red curve. We cannot treat our own liability price as a liquid one as we are the only providers for such but we may employ the CDS and secondary asset market at times to get a read on the stress level to be used in inferring the price as a liability.

We recognize that our own debt trades in the market where one will find both a bid and an ask price for our debt. These prices are bid and ask prices being offered and demanded by other counterparties trading our debt as an asset in their balance sheet. The market spread reflects liquidity considerations between counterparties essentially considering our debt as an asset. We take these prices as reflecting the valuation of our debt as an asset in the market place and all three prices, the bid, the ask and the midquote reflect an asset valuation and from the perspective of this paper they are all some measure of a bid price or asset value. For the valuation as a liability we recommend the use of some specific family of cones of acceptable risks from which we use the asset value to reverse engineer the stress level given an estimate of risk neutral default probabilities extracted from the CDS curve with the liability value being determined from this stress level by pricing the liability to acceptability. The spread between the value as a liability and the asset value should then be taken as a reserve in the $ODOR$ account.

7.2 Example of a calibration for bank data

Additionally we consider a sample of semiannual coupon bonds issued by 6 banks and we price them as assets and liabilities at a variety of stress levels. The bonds are described in Table 6. We price them as of

November 12, 2009. From data on CDS rates for these banks on November 12, 2009 we estimate the default time distribution in the Weibull family with parameters as provided in Table 7. The mean life is additionally displayed. For details on the estimation procedure we refer to Konikov, Madan and Marinescu (2006).

We present in Table 8 the asset and liability value at four stress levels of .25, .5, .75, and 1.0.

In summary we have illustrated in this section how one may use market data to effectively administer the pricing and reserve taking methodologies set out in this paper.

8 Hedging Considerations

This section recognizes that the availability and use of effective hedging strategies would help raise bid prices and lower ask prices with the effect of reducing reserves and increasing profits. Where this is possible, it should be encouraged as one does not wish to advocate unnecessarily high levels of reserves.

If in addition to our assets and liabilities we have some hedging assets available then we wish to consider the best ask and bid prices or liability and asset values after the hedge. This problem is studied in Cherny and Madan (2010) where it is shown that one must first identify the set of risk neutral measures $\mathcal{R}$ with the property that for every zero cost hedging asset with cash flow $\widetilde{H}$ we have

$$E^Q[\widetilde{H}] = 0, \text{ all } \widetilde{H} \in \mathcal{H}, \text{ all } Q \in \mathcal{R}. \tag{56}$$

The liability is then valued at

$$L = \sup_{Q \in \mathcal{R} \cap \mathcal{M}} E^Q\left[\widetilde{L}\right], \tag{57}$$

and the asset is valued at

$$A = \inf_{Q \in \mathcal{R} \cap \mathcal{M}} E^Q\left[\widetilde{A}\right]. \tag{58}$$

As a consequence the bid ask spread is reduced and in fact goes to zero for claims in the span of the liquid hedging assets.

We leave readers to pursue these issues as a subsequent application of Cherny and Madan (2010) but note here that the answer will depend critically on the quantity and quality of the hedging assets employed. With regard to ones own default it is unclear what hedging assets one is going to employ if one does not get into shorting ones own stock or writing down side puts on ones own stock.

9 Conclusion

We apply the theory of pricing to acceptability developed in an operational way by Cherny and Madan (2010) following the construction of indices of risk acceptability in Cherny and Madan (2009) to problems of marking ones own default risk in incomplete markets. This theory was developed precisely for an incomplete markets context and goes back to earlier work by Carr, Geman and Madan (2001). It is observed quite clearly that in agreement with Heckman (2004), assets and liabilities are should not be valued for the pruposes of financial reporting at the same magnitude. Liabilities are marked at ask prices that are larger than the bid prices appropriate for marking assets. The essential intuition in the context of incomplete markets is that markets do not determine prices but they determine price intervals. The need to unwind suggests that assets be marked near the lower end of the set while liabilities be marked near the upper end.

Applying cones of acceptability defined by the concave distortion *minmaxvar* at the stress level of 0.75 it is shown that counterintuitive profitability resulting from credit deterioration is eliminated. The liabilities are also observed to be analytically priced very close to the Heckman recommendation of pricing them as if they were default free. Following Heckman we suggest that the difference between the liability mark and the asset mark be taken as an upfront expense deposited in a special account called the $ODOR$ account for Own Default Operating Reserve. Examples illustrate the variation of the $ODOR$ account in line with ones own credit quality.

Procedures for pricing coupon bonds separately as assets and as liabilities are presented. The methods require access to the default time distribution. We employ quotes from the CDS market to recover these distributions in the Weibull family of densities.

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TABLE 1

PnL impact of credit changes in millions

Quarter	CDS	Marked as Asset	Marked as Liability
	301		
30 Nov. 08	402	1195.7	17.4
31 Mar. 09	296	-1259.7	-18.2
30 Jun. 09	191	-1494.8	-13.7
30 Sep. 09	138	-908.6	-5.1

Table 2

Values when marked as asset and liability in billions

	Asset Marking	Liability Marking
31 Aug. 08	24.590	31.674
30 Nov. 08	23.395	31.657
31 Mar. 09	24.655	31.675
30 Jun. 09	26.149	31.689
30 Sep. 09	27.058	31.694

Table 3

Quarter	ODOR account in billions
31 Aug. 08	7.084
30 Nov. 08	8.262
31 Mar. 09	7.020
30 Jun. 09	5.539
30 Sep. 09	4.636

TABLE 4

Priced at Bid

	a		
c	1.1	1.5	2
10	2725	3480	4274
15	3497	4424	5318
20	4017	5008	5894

TABLE 5

Priced at Ask

	a		
c	1.1	1.5	2
10	7232	7377	7477
15	7416	7514	7564
20	7488	7556	7583

Table 6

Issuer	Coupon	Maturity
JPM	4.75	1-Mar-2015
MS	6.00	28-Apr-2015
GS	5.5	15-Nov-2014
BAC	5.125	15-Nov-2014
WFC	5.0	15-Nov-2014
C	4.875	7-May-2015

Table 7

Weibull Parameters

Ticker	c	a	mean in years
JPM	48.99	1.2599	45.55
MS	35.32	1.1382	33.72
GS	48.08	1.0983	46.42
BAC	28.92	1.1626	27.44
WFC	41.36	1.0832	40.12
C	24.41	1.0335	24.09

Table 8

	.25		.5		.75		1.0	
	Asset	Liab.	Asset	Liab.	Asset	Liab.	Asset	Liab.
JPM	70.73	85.39	60.54	88.12	50.17	89.35	40.41	89.88
MS	60.27	80.16	48.69	84.87	38.06	87.36	28.93	88.61
GS	67.98	84.88	56.95	88.31	46.13	89.94	36.28	90.69
BAC	58.79	80.15	46.82	85.46	36.05	88.34	26.98	89.84
WFC	64.69	83.34	53.19	87.42	42.27	89.47	32.61	90.45
C	48.56	72.83	36.71	79.96	26.87	84.28	19.17	86.77

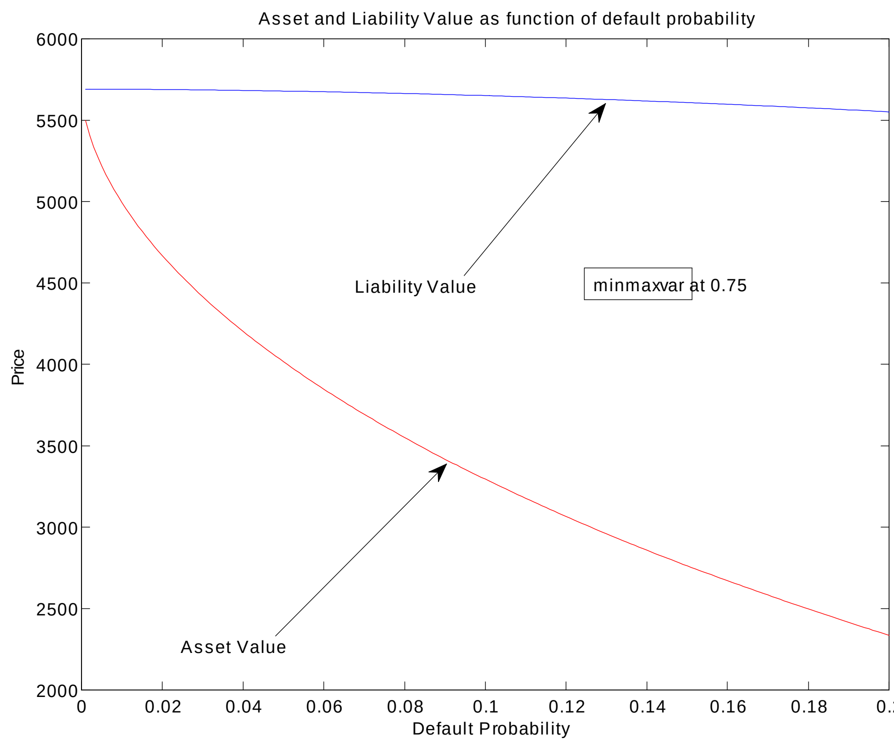

Figure 1: Asset and Liability values of the Heckman example as functions of the default probability when priced to acceptability for minmaxvar at stress level 0.75.

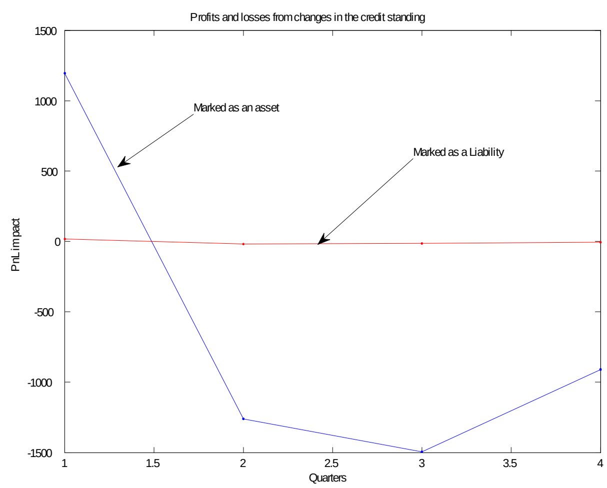

Figure 2: Profits and losses from credit deterioration over a hypothetical year when marked as an asset and when marked as a liability.

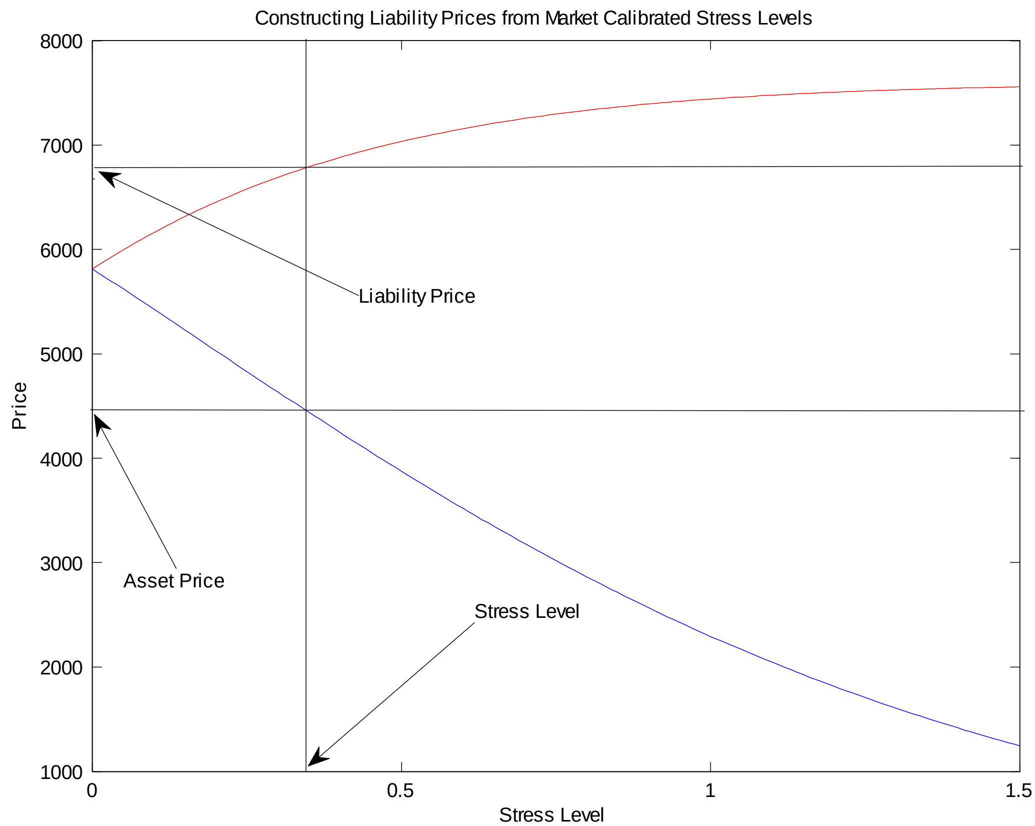

Figure 3: Graph showing how to infer the liability price from the asset price given a default time probability function extracted from the CDS market.