

Accounting to Acceptability: With applications to the pricing of ones own credit risk.

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Abstract

The theory of pricing to acceptability developed for incomplete markets by Cherny and Madan (2009b) is applied to marking ones own default risk. It is observed in agreement with Heckman (2004), that assets and liabilities are not to be priced under fair value accounting principles at the same magnitude. Liabilities are marked at ask prices that are above the asset mark at bid prices. Applying cones of acceptability defined by the concave distortion *minmaxvar* at the stress level of 0.75 it is shown that counterintuitive profitability resulting from credit deterioration is eliminated. Following Heckman we suggest that the difference between the liability mark and the asset mark be taken as an upfront expense deposited in a special account called the *ODOR* account for Own Default Operating Reserve. Procedures for pricing coupon bonds separately as assets and liabilities are described. They employ the default time distribution as calibrated from the CDS market.

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1 Introduction

A perturbing development now frequently reported in the press and in statements by CEOs explaining earnings statements is the adverse effect on revenues of improvements in credit conditions, or the profits being reported as a consequence of a deterioration in ones own credit rating. This situation is a consequence of accounting for ones own credit risk in reporting liabilities with a lower liability being reported when one is less likely to actually make the payment, and hence the resulting profit.

By way of examples we quote from Risk magazine, October 22 2009 as follows. “Morgan Stanley announced a \$757 million profit for the third quarter on October 21, compared with a loss of \$159 million in the second quarter. However, it said it had taken a \$0.9 billion loss as CDS spreads referencing the bank’s own debt narrowed further. Five year senior CDS spreads on the firm moved from 191 basis points to 138bp across the quarter. . . . However in 2008, the bank’s fixed-income business alone enjoyed a profit of \$3.5 billion on its own liabilities due to the bank’s credit spreads ballooning from 98bp to 402bp over the course of the year.”

Philip Heckman (2004) masterfully argued against this practice and pointed out the above counterintuitive consequences of making such credit adjustments. He advocated instead that the liability should always be reported on a going concern basis and be valued as a default free liability, while acknowledging that the lower value obtained by discounting at the higher credit sensitive risky rate was admissible as an asset value. He argued that assets and liabilities are not to be equally valued.

Wallace (2004) offers a rebuttal of Heckman pointing out that modern financial theory recognizes the need for higher returns related to greater risks and two equivalent promises from counterparties with differing capacity to pay are not the same product and hence do not have the same value. However, their value though different on account of the capacity to pay, is the same for each on both sides of the balance sheet by virtue of the law of one price in markets.

We shall show that the problem arises in markets with fading liquidity. In such markets it matters whether assets are being bought from or sold to the market. Transaction prices will differ significantly with the trade direction of parties treating the market as a counterparty. Only when liquidity grows again, do bid-ask spreads converge making the so-called “law of one price” an acceptable approximation. Standard theory, abstracting from market liquidity considerations may seriously misguide decisions and valuations.

There is a well developed financial theory for the pricing and valuation of liquid assets. Here the law of one price prevails, bid ask spreads are zero, and all claims are priced as discounted expectations under an equilibrium pricing measure or probability. However, the market for individual specific credit sensitive debt is too specific to be liquid. The housing market may be liquid but once we get down to the specific market for one’s own house we lose liquidity. It may have traded just once in the last 20 years.

Fair value accounting principles in incomplete markets have to recognize the

consequences of illiquidity embedded in the presence of bid ask spreads. As a consequence, assets when they are sold, go out at the bid price of counterparties and liabilities when reversed, must pay the ask price. What is missing in standard theory and addressed by an emerging theory on market incompleteness, is that assets and liabilities are not equally marked. This equality only applies to liquid assets and they are not under discussion here. What is needed is a theory of bid and ask prices in incomplete markets along with procedures for computing the same.

The story begins with recognizing upfront what one is going to do with residual risk, for incompleteness means that this risk is present and cannot be eliminated. Exact replication using liquid assets is out of the question and the best hedge leaves one exposed to residual risk. The question to be answered is what residual risk will be willingly held by all market participants. One answer is obvious, any residual risk that is always positive to the counterparty, will be willingly held by anyone. These considerations lead to ask prices being the lowest cost of superreplication while bid prices are the highest cost of subreplication. The resulting bid ask spreads are however huge and not relevant for practical considerations.

It is clear that most market participants will take some exposure to loss and cannot insist that the residual is always positive for them. Artzner, Delbaen, Eber and Heath (1999) axiomatized acceptable risks as some convex cone containing the cone of nonnegative cash flows. Based on this formulation for acceptable risks Carr, Geman and Madan (2001) reformulated the problem for constructing bid and ask prices and showed as expected that the spread comes down as we widen the cone of acceptable risks. But there was no effective description of the cone and no way to compute either the bid or the ask price.

The situation changed in Cherny and Madan (2009a). For a static model with a fixed horizon, Cherny and Madan (2009a) related cones of acceptability that depend solely on the probability law of a cash flow, to positive expectations under concave distortions. For such probability law dependent cones, the decision on whether a risk is acceptable or not is by construction determined by its probability distribution. Hence in deciding on the acceptability of a random cash flow X , all we need to know is its probability distribution $F(x) = \Pr(X \leq x)$. Such cones may be constructed from a concave distribution function $\Psi(u)$ defined on the unit interval, used as a distortion of probability, by the condition that X is acceptable just if

$$\int_{-\infty}^{\infty} x d\Psi(F(x)) \geq 0. \tag{1}$$

Cherny and Madan (2009a) go on to construct a parameterized sequence of cones $\Psi^\gamma(u)$ associated with a decreasing sequence of cones \mathcal{A}_γ that start at a half space for $\gamma = 0$ and finish at the non-negative cash flows as γ tends to infinity. The theory of pricing to acceptability using such cones has been employed to compute bid and ask prices in Eberlein and Madan (2009), Madan (2009a), and is set out in detail in Cherny and Madan (2009b). In most applications the

distortion used is termed *minmaxvar* where the parameter γ represents the stress level that increases with γ and for *minmaxvar*,

$$\Psi^\gamma(u) = 1 - (1 - u^{\frac{1}{1+\gamma}})^{1+\gamma}. \quad (2)$$

To shed some light on the issues of pricing ones own credit risk we shall follow Heckman (2004) and Wallace (2004) and limit the discussion to the simplest debt instrument, a 10 year zero coupon, non callable, and not putable bond. The face value is 10000 and it is issued by two companies here denoted U, V with risky discount rates of 7, 12 percent and present values of 5083, 3220 with risk free discount at 5.8% and default free price of 5690. We shall first consider the pricing of assets and liabilities in the absence of hedging assets. We shall briefly comment later on the impact of hedging assets but leave the further pursuit of these details to applications of the results obtained in Cherny and Madan (2009b).

The procedures and principles advocated in this paper address the immediate high priority accounting tasks formulated at the Summit on Financial Markets and the World Economy in Washington November 15 2008. In times of stress illiquidity is enhanced and we advocate that complex instruments and swaps be partitioned into their positive and negative parts with the former being priced as an asset on the left of the balance sheet and the latter being priced as a liability on the right hand side of the balance sheet. There should be no off-balance sheet items whatsoever. Furthermore we detail specific procedures for pricing assets and liabilities and thereby begin to address quantitative accounting rules for balance sheets calibrated to observable and measurable stress levels in markets.

The outline of the rest of the paper is as follows. Section 2 reviews the essentials of acceptability pricing as set out in Artzner, Delbaen, Eber and Heath (1999) and its subsequent development, relating these concepts to the accounting issues being considered here. Section 3 takes up the example studied in Heckman (2004) and Wallace (2004) and shows how surprisingly accurate the advocacy of Heckman was from our perspective. Section 4 applies these methods to redo the financial statements of the major banks through the recent crisis with a view to correcting the counterintuitive remarks we opened with by reversing the profitability of credit deterioration. Section 5 extends the computations from pure discount bonds to coupon bonds. Section 6 comments on the broad relationships between asset and liability pricing, noting in particular that the negative part of an asset is a liability and the mechanism of pricing it as an asset automatically prices the negative part as a liability. Section 7 describes a potential calibration strategy for a stress level appropriate for liability pricing. For this purpose we employ the *CDS* market to extract relevant risk neutral probabilities, by treating this *CDS* market as a liquid market for our promises traded as an asset, by numerous counterparties. Section 8 comments on the effects of hedging assets. Section 9 concludes.

2 Essentials of Pricing to Acceptability

We have observed that the set of risks viewed as random variables X on a probability space (Ω, \mathcal{F}, P) that are acceptable to the general economy are modeled as a cone containing the nonnegative cash flows, as the latter are always acceptable by virtue of being devoid of risk. For purposes of corporate accounting, one is in principle reporting to the external world the state of corporate affairs. The accounting reports are not just a statement evaluating shareholder wealth, but they report as profit what the external world must agree as legitimately withdrawable funds from the enterprise. One therefore has to model the risks the external world is willing to accept.

In the complete markets context of modern finance theory and its liquid assets, this profit is clear and equals the market value of the cash flow accessed. Under the unique ‘pricing’ or so-called ‘risk neutral’ measure Q one merely evaluates

$$E^Q[X],$$

for a zero cost cash flow. If this expectation is positive one undertakes the activity generating X as we then have a positive α adequately compensating risks.

When markets are incomplete, the cone generated by a single measure is too wide. Acceptable risks in Artzner, Delbaen, Eber and Heath (1999) are defined by a convex set of supporting probability measures $Q \in \mathcal{M}$ with the property that X is acceptable just if

$$E^Q[X] \geq 0, \text{ for all } Q \in \mathcal{M}, \text{ or} \quad (3)$$

$$\inf_{Q \in \mathcal{M}} E^Q[X] \geq 0. \quad (4)$$

The class of externally acceptable cash flows is then considerably smaller than the positive α cash flows, and the acceptability requirement is considerably more conservative.

When we come to marking assets and liabilities with nonnegative cash flows to be received or paid out we have to ask what are the externally acceptable terms. For an asset with a random cash flow $\tilde{A} \geq 0$ or a liability with a random cash flow \tilde{L} the prices A, L that are externally acceptable are such that the residuals

$$\tilde{A} - A, L - \tilde{L}$$

are externally acceptable.

It follows immediately that the smallest admissible value for L and the largest admissible value for A is given by

$$L = \sup_{Q \in \mathcal{M}} E^Q[\tilde{L}]$$

$$A = \inf_{Q \in \mathcal{M}} E^Q[\tilde{A}].$$

For the same cash flow its value as an asset is then lower than its value as liability. Consistent with the views of Heckman, asset valuation and the valuation of liabilities are different but related activities. In particular, the cash flow is always worth more as a liability than as an asset. This is none other than the recognition that assets must be sold at bid prices while liabilities are unwound at the ask price.

The question that now arises is, “How do we compute these bid and ask prices or equivalently the marks on the asset and liability side of the balance sheet?”. For this we turn to Cherny and Madan (2009a). Suppose first that acceptability is to be defined completely by the probability law or distribution function $F(x)$ of the risk at hand. Cherny and Madan (2009a) then describe the link between acceptability and concave distortions of the distribution function as expressed in equation (1) for some concave distortion Ψ . The set of supporting measures \mathcal{M} for this set of acceptable risks is all measures Q with density $Z = \frac{dQ}{dP}$ satisfying the condition

$$E^P \left[(Z - a)^+ \right] \leq \Phi(a) = \sup_{u \in [0,1]} (\Psi(u) - ua), \text{ for all } a \geq 0$$

In summary, the condition (1) defines a valid cone of acceptable risks that depend on just a knowledge of the distribution function of the cash flow. We may observe on rewriting the integral in condition (1), assuming that F has a density f , as

$$\int_{-\infty}^{\infty} x \Psi'(F(x)) f(x) dx,$$

that our expectation under concave distortion is also an expectation under a measure change. We note that large losses with $F(x)$ near zero are reweighted upwards by $\Psi'(F(x))$ as Ψ' decreases for any concave distortion. The more concave the distortion the higher the upward reweighting of losses and the more difficult it is to be acceptable.

Cherny and Madan (2009a) go on to propose a sequence of concave distortions indexed by a real number γ that are increasingly more concave with a corresponding decreasing sequence of sets of acceptability. The recommended distortion that we employ in this paper is *minmaxvar* with Ψ^γ as defined in (2)

A simple computation yields the equation for the asset value, for the cone indexed by γ , as

$$A = \int_{-\infty}^{\infty} x d\Psi^\gamma(F_{\bar{A}}(x)) \tag{5}$$

with a computation associated with a simulated set of cash flows sorted into increasing order as $x_1 \leq x_2 \leq \dots x_N$ by

$$C \approx \sum_{j=1}^N x_j \left(\Psi^\gamma \left(\frac{j}{N} \right) - \Psi^\gamma \left(\frac{j-1}{N} \right) \right).$$

For the liability value the comparable result yields

$$L = - \int_{-\infty}^{\infty} x d\Psi^\gamma(F_{-\tilde{L}}(x)).$$

In this case we sort into increasing order a simulation from the law of $-\tilde{L}$, $x_1 \leq x_2 \leq \dots x_N$ and evaluate

$$L = - \sum_{j=1}^N x_j \left(\Psi^\gamma \left(\frac{j}{N} \right) - \Psi^\gamma \left(\frac{j-1}{N} \right) \right).$$

One may show directly from the concavity of Ψ^γ that $L \geq A$ for the same random variable (Madan (2009)).

2.1 Remarks on the base measure

The earlier papers defining acceptability, for example Artzner, Delbaen, Eber and Heath (1999) and its follow up papers up to Cherny and Madan (2009) took the base measure to be the physical or true measure P . We note now that from the perspective of reporting the state of affairs of an entity to the external or general economy this may not be an adequate base measure. This is because a positive expectation that fails to earn sufficient compensation for risk undertaken may not be approved by the external economy. From the perspective of reporting to the external economy we may wish to take as a base measure a particular risk neutral measure with a positive expectation now being a positive alpha trade. Approving all positive alpha trades may be too generous given market incompleteness and hence we may reduce the cone of acceptability by requiring positive expectation with respect to a convex set of measures equivalent to our base risk neutral measure. For notational convenience we still refer to this base measure as the P measure, noting now that it could be taken to be risk neutral.

3 The Heckman example

What is the value of the liability and what is the asset value for promises of companies U, V in the Heckman examples? In present value terms we ask what amount of money L received for the promised payout and invested at the risk free rate makes the residual cash flow acceptable to the external world. In this case the residual cash flow is

$$L(1.058)^{10} - 10000 \times \mathbf{1}_S$$

where S is the survival or non default set. The answer as we have seen, now including time value considerations is

$$L = \frac{1}{(1.058)^{10}} 10000 \sup_{Q \in \mathcal{M}} E^Q[\mathbf{1}_S].$$

Similarly the value as an asset is

$$A = \frac{1}{(1.058)^{10}} 10000 \inf_{Q \in \mathcal{M}} E^Q [\mathbf{1}_S].$$

If we agree that the asset prices are as in the Heckman example, at 5083, and 3220 for U, V respectively then as a liability it is higher.

If we use the *minmaxvar* cone at level γ then as the probability distribution here is a simple one with default probabilities p_U, p_V respectively for U, V we get that

$$\begin{aligned} 5083 &= \frac{1}{(1.058)^{10}} 10000 \times (1 - \Psi^\gamma(p_U)), \\ 3220 &= \frac{1}{(1.058)^{10}} 10000 \times (1 - \Psi^\gamma(p_V)), \end{aligned}$$

and the liability values are respectively

$$\begin{aligned} L_U &= \frac{1}{(1.058)^{10}} 10000 \times \Psi^\gamma(1 - p_U), \\ L_V &= \frac{1}{(1.058)^{10}} 10000 \times \Psi^\gamma(1 - p_V). \end{aligned}$$

If we use *minmaxvar* at level 0.75 as advocated in Madan (2009b) we have

$$\begin{aligned} p_U &= 0.0078 \\ p_V &= 0.1062 \\ L_U &= 5689 \\ L_V &= 5643 \end{aligned}$$

For $\gamma = .5$ the results are

$$\begin{aligned} p_U &= 0.0195 \\ p_V &= 0.1775 \\ L_U &= 5682 \\ L_V &= 5447 \end{aligned}$$

One may graph the value of the asset and liability as a function of the default probability. We present in Figure (1) the graph of both the asset value and the liability value for default probabilities ranging from 0.0010 to 0.2. We observe that the asset value is very sensitive to this probability with the price dropping from around 5500 to around 2500. If the liability were marked the same way a credit deterioration would reflect an enormous profitability associated with this deterioration. When marked to acceptability however the value of the liability barely moves from the risk free level of 5690 down to 5600. The result is very close to the Heckman advocacy of just using the risk free value of 5690.

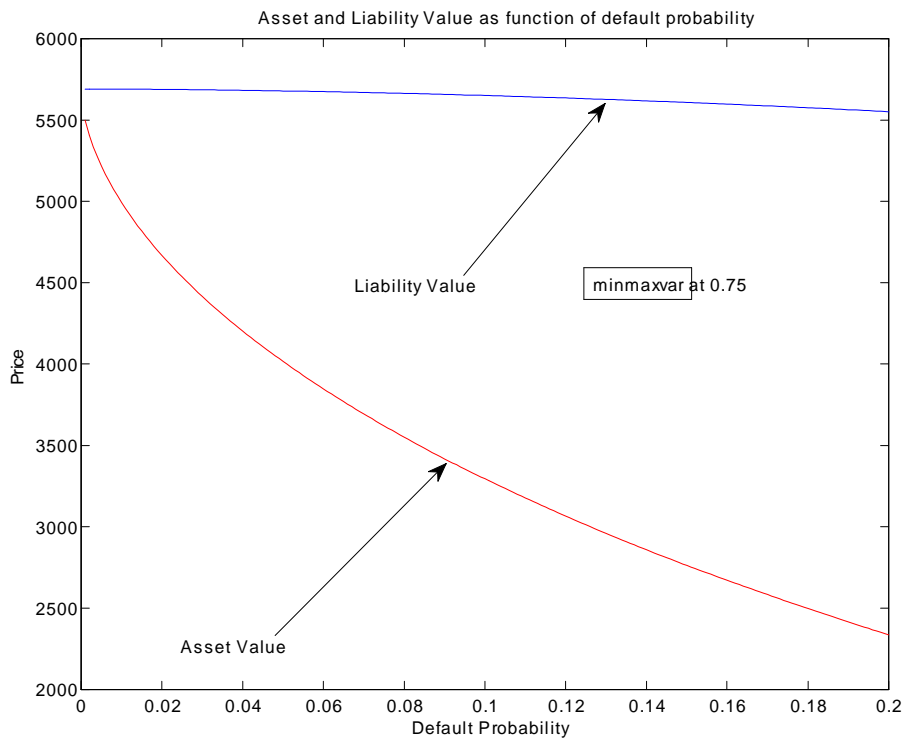


Figure 1: Asset and Liability values as functions of the default probability when priced to acceptability for minmaxvar at stress level 0.75.

There is analytical support for the Heckman advocacy in pricing to acceptability as the liability valuation is

$$L = \frac{1}{(1.058)^{10}} 10000 \times \Psi^\gamma(1-p)$$

Now we may write as an approximation valid for small default probabilities p

$$\Psi^\gamma(1-p) \approx 1 - \Psi^{\gamma'}(1)p + \frac{1}{2}\Psi^{\gamma''}(1)p^2$$

We have advocated the use of distortions that discount large gains down to zero and this requires that $\Psi^{\gamma'}(1) = 0$ and hence to first order in probability $\Psi^\gamma(1-p) \approx 1$ or we use the risk free rate for discounting as advocated by Heckman.

In the case of *minmaxvar* we have

$$\Psi^{\gamma''}(x) = -\frac{\gamma}{1+\gamma} \left[\left(1 - x^{\frac{1}{1+\gamma}}\right)^{\gamma-1} x^{-\frac{2\gamma}{1+\gamma}} + \left(1 - x^{\frac{1}{1+\gamma}}\right)^\gamma x^{-\frac{2\gamma+1}{1+\gamma}} \right]$$

and we see that if we take $\gamma > 1$ then $\Psi^{\gamma''}(1)$ is also zero. For high values of γ the Heckman prescription is absolutely correct. There is some minimal leniency in relaxing the liability value for wider and more tolerant cones for acceptable risk and when default probabilities are high. The sensitivity of the liability value to the default probability is however substantially reduced.

4 PnL impact of credit changes

We consider a 5 year maturity with a face value of 35 billion dollars and an interest rate of 2%. We take credit spreads at quarter ends of 301, 402, 296, 191, and 138 basis points. We price the debt as an asset and as a liability using *minmaxvar* at stress level 0.75 and report the *PnL* impact across four successive quarters in Table 1.

TABLE 1
PnL impact of Credit changes in millions

Quarter	CDS	Marked as Asset	Marked as Liability
	301		
30 Nov. 08	402	1195.7	17.4
31 Mar. 09	296	-1259.7	-18.2
30 Jun. 09	191	-1494.8	-13.7
30 Sep. 09	138	-908.6	-5.1

A graph of the *PnL* impact is presented in Figure (2).

The asset and liability values are presented at the end of each quarter in

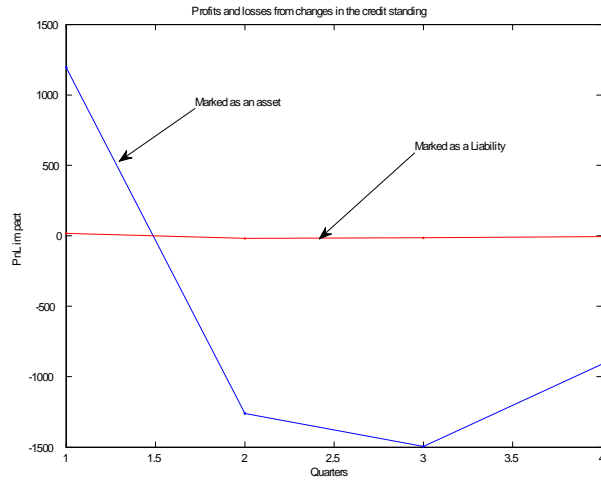


Figure 2: Profits and losses from Credit Deterioration over a hypothetical year when marked as an asset and when marked as a liability.

Table 2.

	Asset Marking	Liability Marking
31 Aug. 08	24.590	31.674
30 Nov. 08	23.395	31.657
31 Mar. 09	24.655	31.675
30 Jun. 09	26.149	31.689
30 Sep. 09	27.058	31.694

The question now arises as to how one should account for the difference between the mark as a liability and the mark as an asset. If a lender of last resort in fact existed one could price the put option and show it as an asset purchased for the expense given by the difference in values. Of course this is not the case, though we agree with Heckman that the difference is an expense but where does it go. A possible location is as a cash reserve against ones own default. This can be an *ODOR* account for Own Default Operating Reserve. In our example as the credit situation changes the *ODOR* reserve varies reflecting the value of the reserve appropriate for own default in line with the current default probability. Table 3 gives the levels of the *ODOR* account at quarter end. The level of this

account represents the level of deterioration in ones own credit quality.

Quarter	ODOR account in billions
31 Aug. 08	7.084
30 Nov. 08	8.262
31 Mar. 09	7.020
30 Jun. 09	5.539
30 Sep. 09	4.636

5 Bid and Ask Prices for coupon bonds

Consider now a sequence of payments in the amount c_i to be made or received at time t_i for a finite sequence of times $t_1 < t_2 < \dots < t_n$. Let the random default time be τ with distribution function

$$P(\tau \leq t) = F(t), \quad t \geq 0.$$

Let us first consider two payments or receipts c_1, c_2 at times $t_1 < t_2$. First we model the bid price as an asset. Let the risk free discount factors be d_1 and d_2 . There are three possible present value payments and they are 0, $c_1 d_1$, $c_1 d_1 + c_2 d_2$. The probabilities are respectively $F(t_1)$, $F(t_2) - F(t_1)$, and $1 - F(t_2)$. The distribution function at the three cash flows is $F(t_1)$, $F(t_2)$, 1. The bid price or price as an asset, A , is

$$A = c_1 d_1 (\Psi^\gamma(F(t_2)) - \Psi^\gamma(F(t_1))) + (c_1 d_1 + c_2 d_2) (1 - \Psi^\gamma(F(t_2))).$$

As a liability the cash flows in increasing order are $-(c_1 d_1 + c_2 d_2)$, $-c_1 d_1$, and 0. The probabilities are now $1 - F(t_2)$, $F(t_2) - F(t_1)$, and $F(t_1)$. The distribution function for the negative of the cash flow is $1 - F(t_2)$, $1 - F(t_1)$, and 1. The ask price or price as a liability, L ,

$$L = (c_1 d_1 + c_2 d_2) \Psi^\gamma(1 - F(t_2)) + c_1 d_1 (\Psi^\gamma(1 - F(t_1)) - \Psi^\gamma(1 - F(t_2))).$$

More generally for n payments we have

$$A = \sum_{j=1}^n \left(\sum_{i=1}^j c_i d_i \right) [\Psi^\gamma(F(t_{j+1})) - \Psi^\gamma(F(t_j))] \quad (6)$$

where $t_{n+1} = \infty$. For a liability we have

$$L = \sum_{j=1}^n \left(\sum_{i=1}^j c_i d_i \right) [\Psi^\gamma(1 - F(t_j)) - \Psi^\gamma(1 - F(t_{j+1}))]. \quad (7)$$

For a specific example let us take a default time distribution in the Weibull family with

$$F(t) = 1 - \exp\left(-\left(\frac{t}{c}\right)^a\right), \quad t \geq 0$$

with five annual payments of 1000, 2000, 500, 700, and 4000 dollars. Let the risk free rates be .01, .0125, .015, .02, .025. The discount curve is .99, .9753, .9560, .9231, and .8825. The price as a risk free set of cash flows is 7594.84.

For three settings for c of 10, 15, and 20 and three settings for a of 1.1, 1.5, and 2.0 we compute using the expressions (6) and (7) the price for this claim as an asset and a liability. The distortion used is again *minmaxvar* at the level 0.75. The results are presented in Table 4 and Table 5 respectively for an asset and a liability.

TABLE 4
Priced at Bid

	a		
c	1.1	1.5	2
10	2725	3480	4274
15	3497	4424	5318
20	4017	5008	5894

TABLE 5
Priced at Ask

	a		
c	1.1	1.5	2
10	7232	7377	7477
15	7416	7514	7564
20	7488	7556	7583

6 General remarks relating bid and ask prices

The random cash flow X to be priced as an asset or a liability could have both positive and negative components. We may also write the cash flow as the difference of its positive and negative components with

$$X = X^+ - X^-$$

where X^+ is an asset and X^- is a liability. We observe in this section that the bid price of X is the bid price of X^+ less the ask price of X^- . Hence X^- is treated like a liability and priced at the ask and X^+ is an asset priced at the bid.

The bid price for X is by construction

$$b(X) = \int_{-\infty}^{\infty} x d\Psi(F_X(x))$$

while the ask price is

$$a(X) = - \int_{-\infty}^{\infty} x d\Psi(F_{(-X)}(x))$$

Consider now the bid price of $(-X)$ and this is

$$\begin{aligned} b((-X)) &= \int_{-\infty}^{\infty} x d\Psi(F_{(-X)}(x)) \\ &= -a(X) \end{aligned} \tag{8}$$

Now the bid price of X^+ is

$$b(X^+) = \int_0^{\infty} x d\Psi(F_X(x))$$

also the ask price of X^- is

$$\begin{aligned} a(X^-) &= - \int_{-\infty}^0 x d\Psi(F_{(-X^-)}(x)) \\ &= - \int_{-\infty}^0 x d\Psi(F_X(x)) \end{aligned}$$

Hence

$$b(X) = b(X^+) - a(X^-).$$

Similarly by applying (8) we learn that

$$\begin{aligned} a(X) &= -b((-X)) \\ &= a((-X)^-) - b((-X)^+) \\ &= a(X^+) - b(X^-) \end{aligned}$$

so when X is a liability its negative part is an asset and it is priced at bid while its positive part is a liability and this is priced at the ask.

In general for swap type contracts we recommend that they be split into their positive and negative parts with the positive part being priced at bid and the negative part priced at ask. The situation arises here of a zero net value swap traded at market that is immediately marked negatively as the bid price reduces the value of the positive part and the ask price raises the value of the negative part. The difference is however not a loss but should be taken as a reserve against possibly disadvantageous future unwinds.

6.1 Aggregation considerations

From the structure of bid pricing seen as an infimum of admissible valuations it is clear that the bid price of a package is greater than the sum of the bid prices of the components. Similarly the ask price of a package is lower than the sum of the ask prices of components. Hence one should aggregate claims into packages and perform bid ask valuations at the level of collections of assets sharing common risk exposures.

7 Liability Pricing via Stress Calibration

We take the position that the market for our own debt as an asset for others may have a relatively liquid market as there are many counterparties for our own debt who could trade directly with us or between themselves. We may take for the bid price some transaction prices in the lower end of observed transactions. Furthermore from the *CDS* market on our own name we may estimate a risk neutral distribution for the default time in the Weibull family to estimate the parameters c, a . One may then employ equations (6) and (7) to construct the asset and the liability price as a function of the stress level.

A graph for our example in section 5 with $c = 10$, $a = 1.25$ is shown in Figure (3). Given the bid or asset price we may read off the stress level along the blue curve and then get the liability price from the red curve. We cannot treat our own liability price as a liquid one as we are the only providers for such but we may employ the *CDS* and secondary asset market at times to get a read on the stress level to be used in inferring the price as a liability.

Additionally we consider a sample of semiannual coupon bonds issued by 6 banks and we price them as assets and liabilities at a variety of stress levels. The bonds are described in Table 6.

Table 6

Issuer	Coupon	Maturity
JPM	4.75	1-Mar-2015
MS	6.00	28-Apr-2015
GS	5.5	15-Nov-2014
BAC	5.125	15-Nov-2014
WFC	5.0	15-Nov-2014
C	4.875	7-May-2015

We price them as of November 12, 2009. From data on CDS rates for these banks on November 12, 2009 we estimate the default time distribution in the Weibull family with parameters as provided in Table 7. The mean life is additionally displayed. For details on the estimation procedure we refer to Konikov, Madan and Marinescu (2006).

Table 7

Weibull Parameters

Ticker	c	a	mean in years
JPM	48.99	1.2599	45.55
MS	35.32	1.1382	33.72
GS	48.08	1.0983	46.42
BAC	28.92	1.1626	27.44
WFC	41.36	1.0832	40.12
C	24.41	1.0335	24.09

We present in Table 8 the asset and liability value at four stress levels of .25,

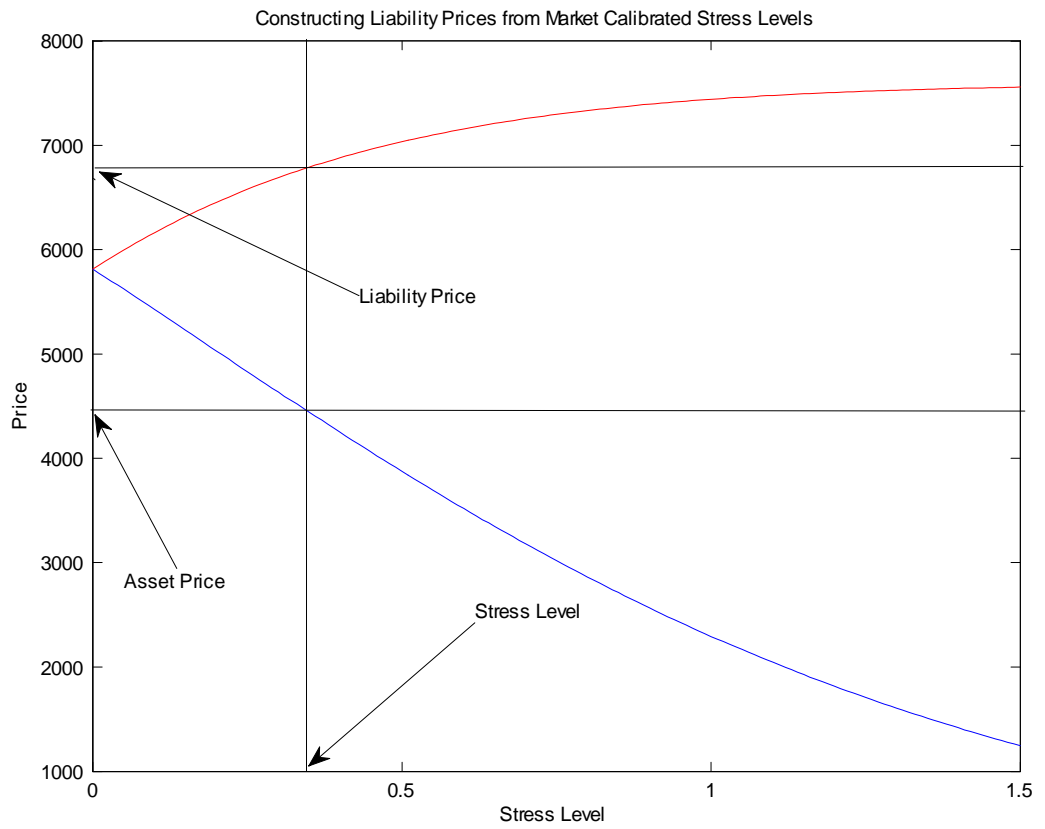


Figure 3: Graph showing how to infer the liability price from the asset price given a default time probability function extracted from the CDS market.

.5, .75, and 1.0.

Table 8

	.25		.5		.75		1.0	
	Asset	Liab.	Asset	Liab.	Asset	Liab.	Asset	Liab.
<i>JPM</i>	70.73	85.39	60.54	88.12	50.17	89.35	40.41	89.88
<i>MS</i>	60.27	80.16	48.69	84.87	38.06	87.36	28.93	88.61
<i>GS</i>	67.98	84.88	56.95	88.31	46.13	89.94	36.28	90.69
<i>BAC</i>	58.79	80.15	46.82	85.46	36.05	88.34	26.98	89.84
<i>WFC</i>	64.69	83.34	53.19	87.42	42.27	89.47	32.61	90.45
<i>C</i>	48.56	72.83	36.71	79.96	26.87	84.28	19.17	86.77

8 Hedging Considerations

If in addition to our assets and liabilities we have some hedging assets available then we wish to consider the best ask and bid prices or liability and asset values after the hedge. This problem is studied in Cherny and Madan (2009b) where it is shown that one must first identify the set of risk neutral measures \mathcal{R} with the property that for every zero cost hedging asset with cash flow \tilde{H} we have

$$E^Q[\tilde{H}] = 0, \text{ all } \tilde{H} \in \mathcal{H}, \text{ all } Q \in \mathcal{R}.$$

The liability is then valued at

$$L = \sup_{Q \in \mathcal{R} \cap \mathcal{M}} E^Q[\tilde{L}],$$

and the asset is valued at

$$A = \inf_{Q \in \mathcal{R} \cap \mathcal{M}} E^Q[\tilde{A}].$$

As a consequence the bid ask spread is reduced and in fact goes to zero for claims in the span of the liquid hedging assets.

We leave readers to pursue these issues as a subsequent application of Cherny and Madan (2009b) but note here that the answer will depend critically on the quantity and quality of the hedging assets employed. With regard to ones own default it is unclear what hedging assets one is going to employ if one does not get into shorting ones own stock or writing down side puts on ones own stock.

9 Conclusion

We apply the theory of pricing to acceptability developed in an operational way by Cherny and Madan (2009b) following the construction of indices of risk acceptability in Cherny and Madan (2009a) to problems of marking ones own default risk in incomplete markets. This theory was developed precisely for an incomplete markets context and goes back to earlier work by Carr, Geman and

Madan (2001). It is observed quite clearly that in agreement with Heckman (2004), assets and liabilities are not to be priced or marked under fair value accounting principles at the same magnitude. Liabilities are marked at ask prices that are larger than the bid prices appropriate for marking assets. The essential intuition in the context of incomplete markets is that markets do not determine prices as points but rather they determine price sets. The need to unwind suggests that assets be marked near the lower end of the set while liabilities be marked near the upper end.

Applying cones of acceptability defined by the concave distortion *minmaxvar* at the stress level of 0.75 it is shown that counterintuitive profitability resulting from credit deterioration is eliminated. The liabilities are also observed to be analytically priced very close to the Heckman recommendation of pricing them as if they were default free. Following Heckman we suggest that the difference between the liability mark and the asset mark be taken as an upfront expense deposited in a special account called the *ODOR* account for Own Default Operating Reserve. Examples illustrate the variation of the *ODOR* account in line with ones own credit quality.

Procedures for pricing coupon bonds separately as assets and as liabilities are presented. The methods require access to the default time distribution. We employ quotes from the *CDS* market to recover these distributions in the Weibull family of densities.

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